

Simple Expressions for Force and Capacitance for a Conductive Sphere near a Conductive Wall

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Abstract—A charged conducting particle near a conducting plane experiences a force that draws it toward the plane. When the particle is far away, the force depends only on the net charge and separation. As the particle approaches, however, the force will also depend on the size of the particle, and whether it is held at constant charge or constant potential. The exact formulas for these forces are quite complicated, but their results can be approximated within a few percent by relatively simple expressions that are much more useful for practical work. These formulas are given and compared to earlier, less accurate approximations.

I. INTRODUCTION

The force on a charged particle near a wall plays a key role in many applications of electrostatics. Xerographic copiers and printers, for example, depend heavily on electrostatic attraction and adhesion to develop and transfer images [1]. A more unpleasant effect is the adhesion of dust to sensitive solar panels, which has been cited as a major problem for unmanned missions to Mars [2]. Another area of concern involves the behavior of microscopic tips near a flat surface, as might occur in the operation of atomic force microscopes or in MEMS devices. In these applications, the voltage, rather than the charge, is usually constrained.

A common way to model these situations is to consider the interaction between a sphere and a flat plane. With this situation, it is possible to obtain an analytical solution for the electrostatic fields using either the method of images or separation of variables. Both methods reduce to simple formulas that are accurate when the sphere is far from the wall. Close to the wall, however, the solutions are infinite series of terms that converge very slowly, often requiring thousands or even millions of terms to obtain an accurate solution.

To avoid the complex calculations, and to provide formulas that are easy to use in practical engineering, several approximations have been put forth in the past. These are usually based on the results that are valid when the sphere is far from the wall. The basic formulas for this case are, for the capacitance of an isolated sphere of radius R in a dielectric medium with

permittivity ϵ .

$$C_\infty = 4\pi\epsilon R \quad (1)$$

and for the force, using the method of images

$$F_\infty = \frac{Q^2}{16\pi\epsilon h^2} = \frac{\pi\epsilon R^2 V^2}{h^2} \quad (2)$$

The two expressions for force give the same result, but are individually useful when either the charge or the voltage on the sphere is constrained. Their equivalence is apparent if the relation between charge and voltage, $Q = CV$, is substituted.

These approximations are based on a point charge, but it is well known that the charge on a conductive sphere is distributed around its surface, and can move in response to external objects. To make these formulas more accurate when the sphere is close to the wall, the equivalent point charge is usually placed somewhere on or inside the sphere. Although this helps somewhat, it is not possible to account for the forces when the sphere separation is on the order of its radius or less, as will be shown in this article.

As an alternate to these ad-hoc corrections, the exact field solutions will be used to calculate the capacitance and force for the sphere at any distance from the wall. Although they require a very long summation, they will be approximated in this article by simple formulas that should be adequate for most engineering and design work.

II. DESCRIPTION

A sphere near a wall can be represented directly in the bispherical coordinate system shown in Figure 1. In this system, the surfaces of constant η represent spheres centered on the z -axis. The surface $\eta = 0$ represents the horizontal x - y plane (or a sphere with infinite radius). As η increases, the spheres become smaller and farther from the wall. Surfaces of constant θ are perpendicular to the spheres, and serve as the coordinate to locate a position on the sphere or wall.

The relation between the rectangular and bispherical coordinate systems is given as [3]

$$\begin{aligned} x &= \frac{a \sin \theta \cos \psi}{\cosh \eta - \cos \theta} \\ y &= \frac{a \sin \theta \sin \psi}{\cosh \eta - \cos \theta} \\ z &= \frac{a \sinh \eta}{\cosh \eta - \cos \theta} \end{aligned} \quad (3)$$

where ψ is the angle around the z -axis, and a is a constant related to the scale of the coordinates. Using these relations allows us to relate the size and spacing of any sphere to its bipolar coordinates. In particular, for the spherical conductor at $\eta = \eta_0$, we find that the constant a is given by

$$a = R \sinh \eta_0 \quad (4)$$

and the spacing from the sphere to ground is

$$h = \frac{a \sinh \eta_0}{\cosh \eta_0 + 1} = \frac{R \sinh^2 \eta_0}{\cosh \eta_0 + 1} = R(\cosh \eta_0 + 1) \quad (5)$$

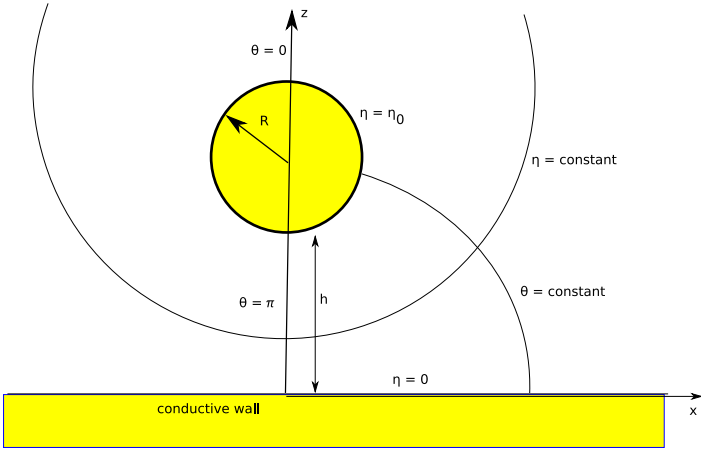


Fig. 1. Definition of the bipolar geometry for a sphere near a plane wall

Solving these equations gives the coordinate of the spherical conductor as

$$\eta_0 = \text{acosh}(1 + h/R) = \text{acosh}(1 + \xi) \quad (6)$$

The ratio $\xi = h/R$ is a single parameter that describes the spacing relative to the size of the conductive sphere.

Integrals and derivatives in the bipolar coordinate system require a knowledge of the metric coefficients that account for the curvature of the coordinate surfaces. These coefficients are given by

$$\begin{aligned} g_{\theta\theta} &= \frac{(R \sinh \eta_0)^2}{(\cosh \eta - \cos \theta)^2} \\ g_{\psi\psi} &= \frac{(R \sinh \eta_0)^2 \sin^2 \theta}{(\cosh \eta - \cos \theta)^2} \end{aligned} \quad (7)$$

and will be used later in calculations of charge and electric fields.

III. FORCE ON THE SPHERE

Both the sphere and the wall are conductors, and the space between them is insulative, so we can use energy functions to find the force on the sphere, as described in standard references [4, 5]. As the sphere moves, two limiting cases can be distinguished. In one, the voltage on the sphere remains constant, which would occur if an electrical connection was maintained between the sphere and an external voltage source. In the other case, the total charge on the sphere remains constant, corresponding to an isolated charged particle. The force expressions for these two cases are different.

A. Force for constant voltage

If the voltage on the sphere is constant during its motion, the appropriate force expression is given by

$$F_V = \left. \frac{dW'}{dh} \right|_v = \frac{1}{R} \left. \frac{dW'}{d\xi} \right|_v \quad (8)$$

using the definition of $\xi = h/R$. The energy function required here is the electrostatic coenergy, W' , given by

$$W' = \frac{1}{2} CV^2 \quad (9)$$

where C is the capacitance between the sphere and the wall, which varies with the size and spacing of the sphere.

The derivative is taken while the voltage is held constant, so the force on the sphere is given by

$$F_V = \frac{V^2}{2R} \frac{dC}{d\xi} \quad (10)$$

Thus to find this force, we must first find the capacitance as a function of spacing.

B. Force for constant charge

The second case requires a different force expression, given by

$$F_Q = - \left. \frac{dW}{dh} \right|_q = - \frac{1}{R} \left. \frac{dW}{d\xi} \right|_q \quad (11)$$

Here the appropriate energy function is the electrostatic energy, W , which is written as

$$W = \frac{Q^2}{2C} \quad (12)$$

In this case, the derivative is taken while the charge is held constant, so only the variation in capacitance is needed. The force is then given by

$$F_Q = \frac{Q^2}{2RC^2} \frac{dC}{d\xi} \quad (13)$$

Notice that this force expression has a different form, since the capacitance appears in the denominator as well as in the derivative. This difference guarantees that the force will behave differently for isolated and connected spheres.

IV. CAPACITANCE

For both force expressions, the capacitance must be known as a function of spacing between sphere and wall. Since the capacitance is defined as

$$C = \frac{Q}{V} \quad (14)$$

we need only find the charge on the sphere as a function of its voltage. The charge within any surface of constant η is given by Gauss' Law as an integral over the entire surface

$$Q = \oiint \varepsilon E_\eta \sqrt{g_{\theta\theta}g_{\psi\psi}} d\theta d\psi \quad (15)$$

where E_η is the normal electric field out of the surface.

Since there is no charge in the space between the sphere and the wall, the integral surface can be placed anywhere between them. It is perhaps simplest to choose the surface at the ground plane ($\eta = 0$). In this case the charge integral becomes, after substituting the values of the metric coefficients from Equation 7.

$$Q = -2\pi\varepsilon R^2 \sinh^2 \eta_0 \int_0^\pi E_G(\theta) \frac{\sin \theta}{(\cosh \eta - \cos \theta)^2} d\theta \quad (16)$$

where E_G is the normal electric field along the ground. The minus sign appears because the field is defined on inner side of surface enclosing the sphere

A. Electric field

The electric field on the grounded wall can be obtained by solving the electrostatic field equations in the bispherical coordinate system to obtain the potential, and then differentiating. The potentials on the wall and the sphere are given by

$$\begin{aligned} \Phi(\eta = 0, \theta) &= 0 \\ \Phi(\eta = \eta_0, \theta) &= V \end{aligned} \quad (17)$$

In addition, the potential is independent of the angle ψ due to symmetry around the z -axis, and must have the same value if θ wraps around,

$$\Phi(\theta) = \Phi(\theta \pm 2\pi) \quad (18)$$

Under these conditions, the potential can be solved by separation of variables [3] to give an infinite series of the form

$$\Phi(\eta, \theta) = V \sum_{n=0}^{\infty} \phi_n(\eta, \theta) \quad (19)$$

where

$$\phi_n(\eta, \theta) = \frac{2\sqrt{2}}{e^{(1+2n)\eta_0} - 1} \sqrt{\cosh \eta - \cos \theta} P_n(\cos \theta) \sinh[(n+1/2)\eta] \quad (20)$$

is the n^{th} term in the series. In this and the following work, a lower case version of a variable indicates that it is dimensionless, and normalized to the situation in which the sphere is very far from the wall.

In bispherical coordinates, the electric field is given by

$$E_\eta = -\frac{\cosh \eta - \cos \theta}{R \sinh \eta_0} \frac{\partial \Phi}{\partial \eta} \quad (21)$$

Substituting the solution for the potential and evaluating on the ground plane gives the electric field there as

$$E_G(\theta) = -\frac{\sqrt{2}V}{R} (1 - \cos \theta)^{3/2} \operatorname{csch}(\eta_0) \sum_{n=0}^{\infty} \frac{(1+2n)P_n(\cos \theta)}{e^{(1+2n)\eta_0} - 1} \quad (22)$$

Substituting this field expression into Equation 16 gives the total charge as

$$Q = \sum_{n=0}^{\infty} Q_n \quad (23)$$

where the n^{th} charge term is

$$Q_n = 2\sqrt{2}\pi\epsilon R V \frac{(1+2n)\sinh \eta_0}{e^{\eta_0+2\eta_0 n} - 1} \int_0^\pi \frac{P_n(\cos \theta) \sin \theta}{(1 - \cos \theta)^{1/2}} d\theta \quad (24)$$

After carrying out the integrals, simplifying, and dividing by V we get the capacitance of the sphere near the wall as

$$C = 4\pi\epsilon R \sum_{n=0}^{\infty} c_n \quad (25)$$

where

$$c_n = \frac{2\sinh \eta_0}{e^{(1+2n)\eta_0} - 1} \quad (26)$$

It is often more convenient to express the capacitance in terms of physical quantities like the size and spacing of the sphere. The only variable in the capacitance term is η_0 , which can be expressed in terms of ξ , the ratio of spacing to radius, by using Equation 6 to replace η_0 , giving

$$c_n(\xi) = \frac{2\sqrt{\xi(2+\xi)}}{e^{(1+2n)\operatorname{acosh}(1+\xi)} - 1} \quad (27)$$

B. Limits and approximations

Limit of a distant sphere

When the sphere is very far away compared to its size ($\xi \rightarrow \infty$ or $\eta_0 \rightarrow \infty$), the terms simplify. The 0th term is

$$c_0(\eta_0) = \frac{2\sinh \eta_0}{e^{\eta_0} - 1} \quad (28)$$

and in limit of large η_0 (isolated sphere) it becomes

$$c_0(\eta_0 \rightarrow \infty) = \frac{2(1/2)e^{\eta_0}}{e^{\eta_0}} = 1 \quad (29)$$

Under the same conditions, all the higher order terms approach zero,

$$c_n(\eta_0 \rightarrow \infty) = \frac{1}{e^{2n\eta_0}} \rightarrow 0 \quad (30)$$

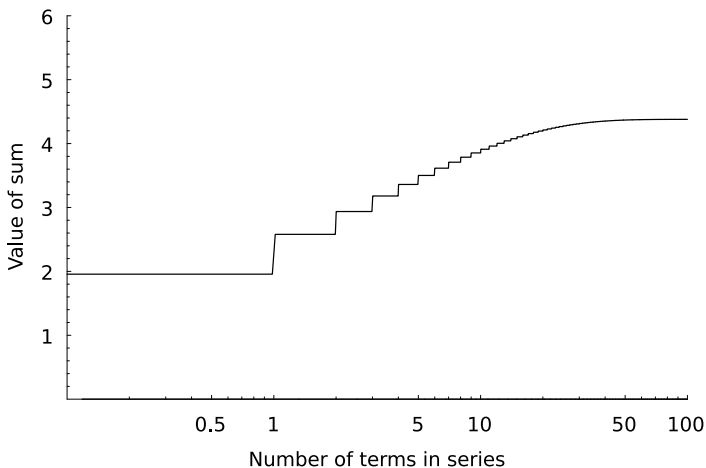


Fig. 2. Partial sums of the normalized capacitance increase as the number of terms increases, but eventually reach a limit ($\xi = 0.001$).

as a result of the term $e^{2n\eta_0}$ in the denominator. In the limit of a distant sphere, therefore, the capacitance approaches the value for an isolated sphere, as it should.

Note that the convergence depends on the exponential term in the denominator becoming large, which implies that $n\eta_0 \gg 1$. When the sphere is very close to the wall, $\eta_0 \rightarrow 0$, and the terms will not decrease in value until $n \gg 1/\eta_0$. This slow convergence at close spacing is the main problem in using traditional methods for studying the electrostatics of a sphere near a wall.

Arbitrary spacing

Although the sum converges slowly, it does in fact converge. Figure 2 shows the successive values for the normalized capacitance for the case where $\xi = h/R = 0.001$ as the number of terms is increased from the 0th to include up to $n = 100$. Each additional term adds to the sum, but in decreasing amounts, until finally the sum appears to level off at a limiting value. Thus a few terms gives good accuracy when the sphere is far from the wall, but many more are needed when it is close.

In the work described here, the required number of terms for good accuracy was determined by comparing successively longer sums. If an increase in the number of terms by 50% did not change the calculated capacitance by 0.01%, the sum was considered converged. This test was performed on the fly for each value of ξ , and required up to 10^6 terms for some points at very close spacing.

A plot of the capacitance as a function of ξ is shown in Figure 3. When the sphere is far from the wall, the capacitance approaches unity (the isolated sphere limit), and it increases as the sphere moves closer. Note that the increase is rather slow, taking 8 decades in spacing (h/R) to increase by a factor of 10.

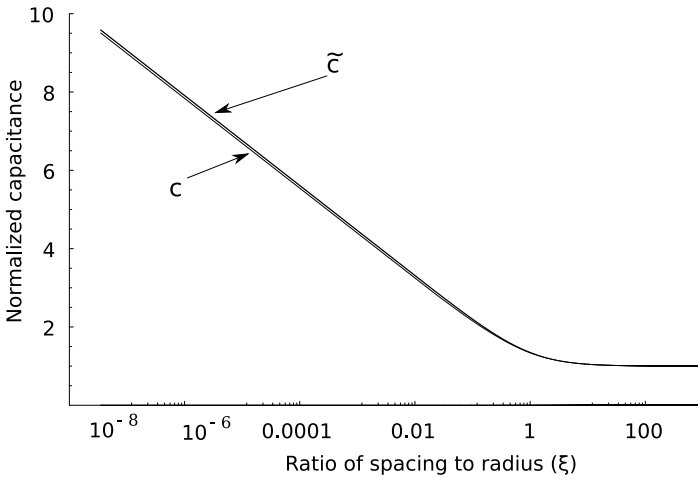


Fig. 3. Capacitance increases slowly as the sphere approaches the wall. Both exact and approximate expressions are shown.

Approximate expression

An analytical expression that involves a million-term sum is not convenient for everyday use, so simpler expressions for the capacitance were investigated. After some trial and error using various fitting functions, it was discovered that the capacitance could be described fairly accurately by an expression of the form

$$\tilde{C} = 4\pi\epsilon R\tilde{c} \quad (31)$$

where the normalized approximate capacitance is

$$\tilde{c} = 1 + \frac{1}{2} \log(1 + 1/\xi) \quad (32)$$

where the tilde indicates that this is an approximation to the actual value. The approximate function is also plotted in Figure 3, where it is clear that it is a very good approximation to the more series expression.

It is easier to see the agreement between the two functions by plotting the difference between them, normalized to the correct value $(\tilde{c} - c)/c$ versus the spacing, as in Figure 4. The error is very small when the spacing is large, and also when the spacing is very small. The largest errors are found in the mid-range of spacing, but are no more than about 2%, even at the worst. This appears to be an acceptable level for routine engineering calculations.

V. FORCE FOR CONSTANT VOLTAGE

Now that the capacitance has been determined as a function of spacing, we can proceed with evaluating the force expressions for constant voltage and constant charge.

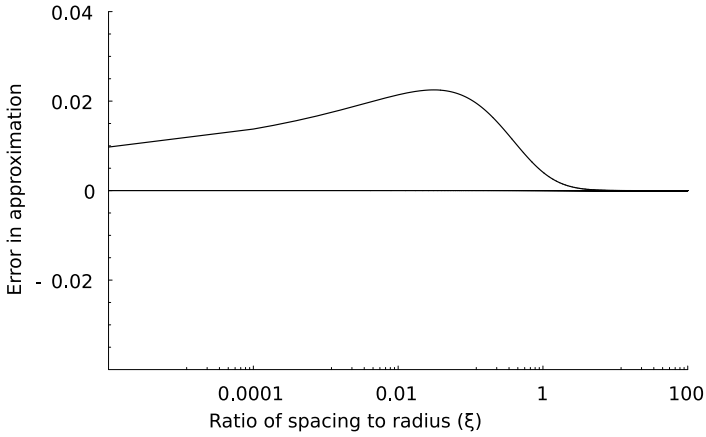


Fig. 4. Fractional error of the approximation to the capacitance

A. Force expressions

When its voltage is held constant, the force on the sphere is given by substitution the capacitance into Equation 10 to obtain

$$F_V = 2\pi\epsilon V^2 \frac{dc}{d\xi} \quad (33)$$

or normalizing to the force when the sphere is very distant

$$f_V = \pi\epsilon R V^2 f_v \quad (34)$$

where the normalized force is

$$f_v = 2 \frac{d}{d\xi} \sum_{n=0}^{\infty} c_n = \sum_{n=0}^{\infty} \frac{d}{d\xi} \left(\frac{2\sqrt{\xi(2+\xi)}}{e^{(1+2n)\operatorname{acosh}(1+\xi)} - 1} \right) \quad (35)$$

Carrying out the differentiations for each term gives the normalized force as

$$f_v = -2 \sum_{n=0}^{\infty} \frac{1 + \xi + e^{(1+2n)\operatorname{acosh}(1+\xi)} \left(-1 - \xi + (1+2n)\sqrt{\xi}\sqrt{2+\xi} \right)}{\left(-1 + e^{(1+2n)\operatorname{acosh}(1+\xi)} \right)^2 \sqrt{\xi}\sqrt{2+\xi}} \quad (36)$$

B. Limit of a distant sphere

When the sphere is far from the wall, the force expression can be expanded in terms of the spacing as

$$f_v = \frac{1}{\xi^2} - \frac{1}{\xi^3} + \frac{3}{4\xi^4} - \frac{25}{16\xi^6} + O\left(\frac{1}{\xi}\right)^{\frac{13}{2}} \quad (37)$$

The leading term gives an inverse square law for force, as expected. The next term is negative, indicating that the force will not increase as strongly as the sphere is moved closer.

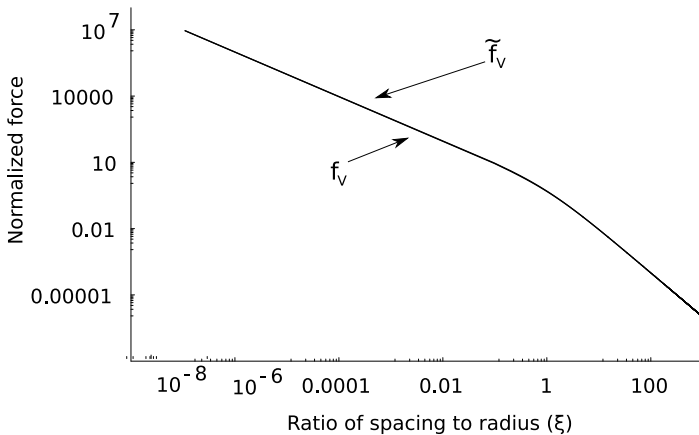


Fig. 5. Force on a sphere at constant voltage increases more slowly near the wall. Both exact and approximate formulas are shown.

C. Arbitrary spacing

The force for constant voltage is plotted over a wide range of spacings in Figure 5. As with the capacitance, a large number of terms must be considered when the gap is small, and the convergence test described above for capacitance was also used for this and the following force calculations.

As the sphere approaches, the force increases, but not as fast as for an inverse square law. Examination of the curve indicates that the increase is proportional to the inverse of the spacing.

D. Approximate expression

A simpler approximate expression for the force can be constructed by using the approximation to the capacitance (Equation 32) instead of the infinite series. In this case, the approximate force is given by

$$\tilde{f}_v = 2 \frac{d\tilde{c}}{d\xi} = 2 \frac{d}{d\xi} [1 + 1/2 \log(1 + 1/\xi)] = \frac{1}{\xi + \xi^2} \quad (38)$$

For large spacing, this expression approaches the inverse square form, just as the exact formula does. At close spacing, it approaches the reciprocal of the spacing, which agrees with the apparent behavior of the exact sum.

The approximate force has been plotted in Figure 5 alongside the exact expression. Comparison of the two shows that the approximation is quite close to the exact expression over the entire range.

The accuracy of the approximation can be better appreciated by examining the relative error, given by $(\tilde{f}_v - f_v)/f_v$ which is plotted in Figure 6. The agreement is best at the extremes of spacing, and worst in the middle, where the approximation always overestimates the force. At the worst, the error is on the order of 5%.

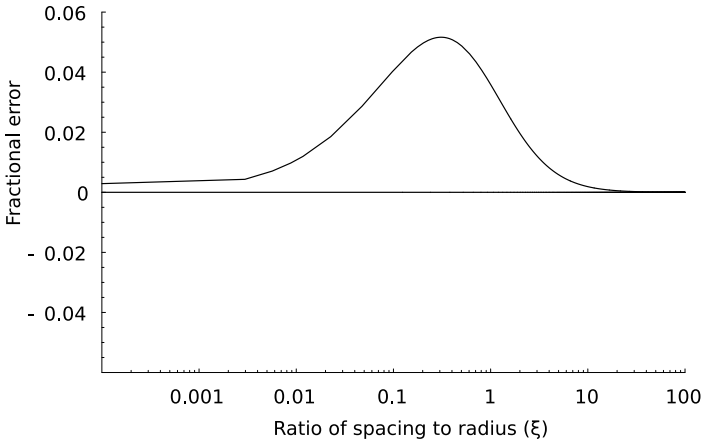


Fig. 6. The fractional error in the approximation to the voltage-constrained force

VI. CONSTANT CHARGE AND ENERGY

A. Force expressions

When the charge is held constant on the sphere, the force expression of Equation 13 takes the form

$$F_Q = \frac{Q^2}{16\pi\epsilon R^2} \frac{2}{c^2} \frac{dc}{d\xi} = \frac{Q^2}{16\pi\epsilon R^2} f_Q \quad (39)$$

where the normalized force is given by

$$f_Q = \frac{2}{c^2} \frac{dc}{d\xi} \quad (40)$$

Combining the derivative term (which has appeared earlier in Equation 36), substituting the sum for the capacitance in the denominator, and simplifying gives the normalized force as

$$f_Q = \frac{\sum_{n=0}^N \left(\frac{1 + \xi + e^{(1+2n) \operatorname{acosh}(1+\xi)} (-1 - \xi + (1+2n) \sqrt{\xi} \sqrt{2+\xi})}{(-1 + e^{(1+2n) \operatorname{acosh}(1+\xi)})^2} \right)}{\xi^{\frac{3}{2}} (2 + \xi)^{\frac{3}{2}} \left(\sum_{n=0}^N \frac{1}{-1 + e^{(1+2n) \operatorname{acosh}(1+\xi)}} \right)^2} \quad (41)$$

This is more complicated than the constant-voltage force, since there are infinite sums in both numerator and denominator, both of which have to be summed independently to obtain the result.

B. Limit of distant sphere

When the charged sphere is far from the wall, the force can be expanded in a series around infinity to give

$$f_Q = \frac{1}{\xi^2} - \frac{2}{\xi^3} + \frac{3}{\xi^4} - \frac{7}{2\xi^5} + \frac{5}{2\xi^6} + O\left(\frac{1}{\xi}\right)^7 \quad (42)$$

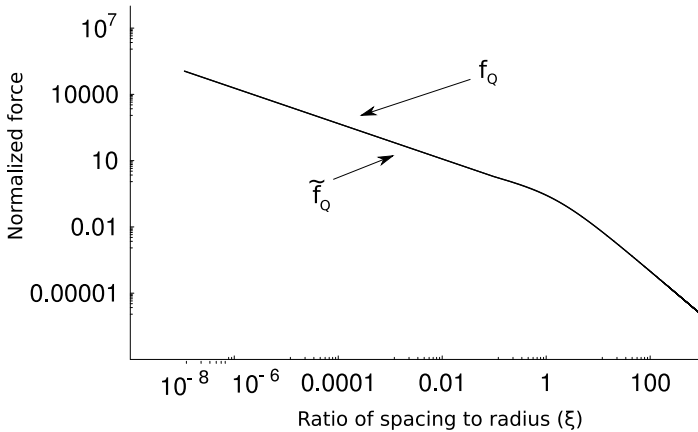


Fig. 7. Force on a sphere with constant charge increases even more slowly near the wall. Both exact and approximate are shown.

As for the constant-voltage force, the force follows an inverse-square law at great distances, but rises more slowly when the sphere approaches the wall.

C. Close sphere

The force for a range of positions was calculated from the exact expression of Equation 41, using the same convergence test described above. A plot of the force versus spacing is shown in Figure 7. Examination of the curve for close spacing reveals that it does not follow a simple power-law expression, unlike the constant-voltage curve.

D. Approximate expression

The calculation of the constant-charge force involves two infinite series. Each of them, like the capacitance and constant-voltage force, requires a very large number of terms when the sphere is close to the wall. We can again use the approximate capacitance of Equation 32 to find a simple expression for the force as

$$\tilde{f}_Q = \frac{1}{\tilde{c}^2} \frac{d\tilde{c}}{d\xi} = \frac{1}{(\xi + \xi^2) \left(1 + \frac{1}{2} \log(1 + 1/\xi)\right)^2} \quad (43)$$

This approximate force expression has been plotted alongside the exact expression in Figure 7, where it is apparent that it gives good agreement.

The fractional error in the approximation $(\tilde{f}_q - f_q) / f_q$ is plotted in Figure 8. The agreement is good for very large and very small spacings, and worse in the middle. Unlike the constant-voltage case, the error here can be positive or negative, but it is less than 3% everywhere.

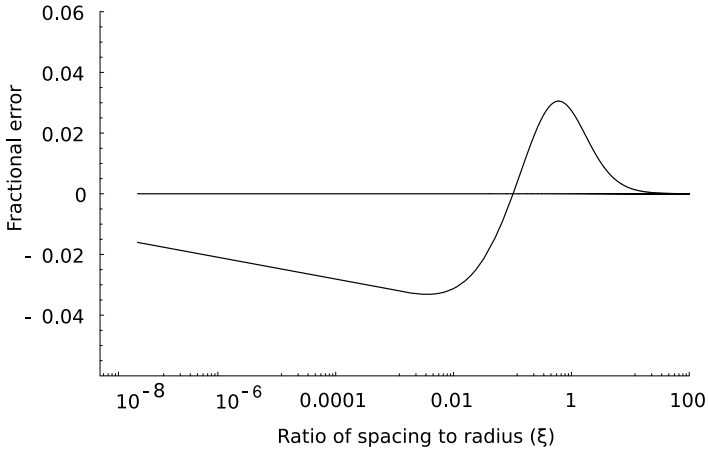


Fig. 8. Error in the approximation to the charge-controlled force

VII. DISCUSSION

Now that we have obtained relatively simple expressions for the force on a sphere near a wall, we can use them to examine some practical questions without requiring extensive computations. As an example, consider some of the earlier approximations that have been used when the attraction and adhesion of charged particles have been discussed.

One of the most used approximations for a charged particle assumes that the charge is concentrated at the center of the particle. This is certainly a good approximation when the sphere is far from the wall, because symmetry ensures that the field outside an isolated charged sphere is equivalent to the field of a single charge placed at the center. In this case, we can use the image force given in Equation 2. In terms of the geometry of Figure 1, the distance from the charge to the wall is $h + R$ instead of h , so the image force expression now becomes

$$F_{\text{center}} = \frac{Q^2}{16\pi\epsilon(h+R)^2} = \frac{Q^2}{16\pi\epsilon R^2} f_{\text{center}} \quad (44)$$

where the normalized force on a centralized charge is

$$f_{\text{center}} = \frac{1}{(1+\xi)^2} \quad (45)$$

One objection to the central-charge approximation is that it does not account for the tendency of charge to re-distribute themselves in the presence of a nearby object. As the sphere approaches the wall, for example, the charge will tend to migrate to the edge of the sphere that is closer to the wall, drawn by the attraction of the image charges in the wall. Since the charge is closer to its image, the force of attraction will be stronger. One way to account for this effect is to assume that all the charge is located at the point of the sphere that is closest to the wall, again assuming that the dielectric constant of the sphere is not important. Using the image

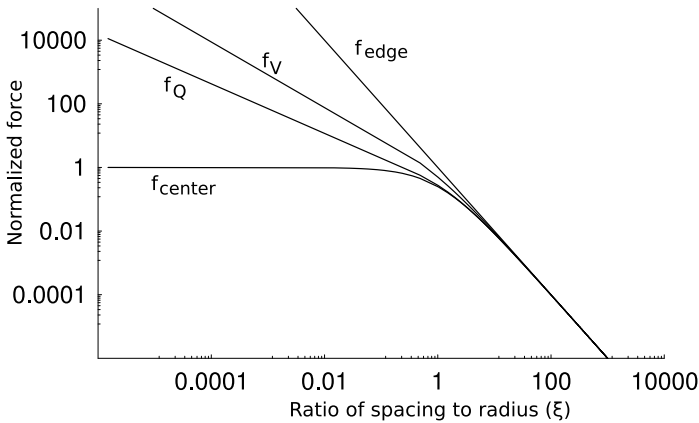


Fig. 9. Comparison of new approximations with two older approximations

force equation gives

$$F_{\text{edge}} = \frac{Q^2}{16\pi\epsilon h^2} = \frac{Q^2}{16\pi\epsilon R^2} f_{\text{edge}} \quad (46)$$

where the normalized force is

$$f_{\text{edge}} = \frac{1}{\xi^2} \quad (47)$$

To see how these earlier approximations compare with those developed in the current paper, we can plot all of them as a function of spacing, as shown in Figure 9. The immediate impression from this figure is that both the central and edge approximations are in error by orders of magnitude when the sphere is very close to the wall. For most applications, this is the crucial regime, since the forces that lead to attraction and attachment are largest when the particle is close to the wall.

If the charge is assumed to be concentrated at the center, the force reaches a limiting value that underestimates the actual force, and completely neglects the increase caused by the migration of charge through the sphere in the direction of the image charge in the wall. This is consistent with experimental results [6, 7] in which electrostatic adhesion forces are at least an order of magnitude larger than the values predicted by the central charge approximation.

The edge approximation is equally bad, since it greatly overestimates the attractive force. By assuming that all the charge is concentrated at a single point, it neglects the spreading of charge over a relatively large area of the sphere when it is close to the wall. Both of the approximations, however, are quite good when the sphere is far from the wall.

When the sphere is not too close to the wall (say, $\xi > 0.3$), and carries a fixed amount of charge, the central approximation gives a noticeably better agreement than the edge approximation, as shown by the two lower curves in the Figure 9. This is interesting because the edge approximation has always been less popular than the central approximation in the past. In general, however, neither of the older approximations is valid when the sphere is close to the wall.

The figure also shows how the constant-voltage and constant-charge forces differ. The force with constant charge is always weaker (by about an order of magnitude) than when the voltage is held constant, although both are closer to each other than to either of the older approximations. When the sphere is close to the wall, the constant-voltage force is stronger because additional charge can flow onto the sphere as it approaches the wall and its capacitance increases. At the same time, the charge is spreading out over the facing surfaces, giving a slower increase in force compared to a point charge.

VIII. CONCLUSION

Approximate formulas for the practical evaluation of capacitance and force for a charged sphere near a wall have been developed. For capacitance, the formula is

$$C \approx 4\pi\epsilon R \left[1 + \frac{1}{2} \log(1 + 1/\xi) \right] \quad (48)$$

where $\xi = h/R$ is the ratio of spacing to radius. When the sphere is held at a constant potential, the force is

$$F_V \approx \pi\epsilon V^2 \left[\frac{1}{\xi + \xi^2} \right] \quad (49)$$

whereas the force when the charge on the sphere is constant is

$$F_Q \approx \frac{Q^2}{16\pi\epsilon R^2} \left[\frac{1}{(\xi + \xi^2) \left(1 + \frac{1}{2} \log(1 + 1/\xi)\right)^2} \right] \quad (50)$$

All of these formulas are valid within about 4% over the entire range of separations, with the best accuracy occurring when the sphere is very far from or very close to the wall.

Other approximations, based on concentrating the charge at some point on the sphere, are much worse when the sphere is close to the wall, although they may be useful when the sphere is separated by 10 or more radii.

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