# Pulsar Rotational Stability

- Pulsars are very good flywheels.
  - Large mass and rapid spin rate means very large angular momentum.
  - Large external torques are required to alter the spin rate appreciably.
- Pulsars are energetic objects and most are powered from their rotational kinetic energy.
  - Pulsars are observed to spin down with time.
  - Energy goes into magnetic dipole radiation, relativistic particles and high-energy radiation.

# Clocks

- A clock can be made from anything that exhibits periodic oscillations.
  - Good clocks need very stable oscillators.
  - Intrinsic frequency stability due to random changes is quantified by the Allan variance.
  - Oscillators can also suffer from systematic instabilities – e.g. frequency drift due to temperature changes in the environment.
- Pendulums have been replaced by quartz crystals and atomic resonators.

# Pulsars as Clocks

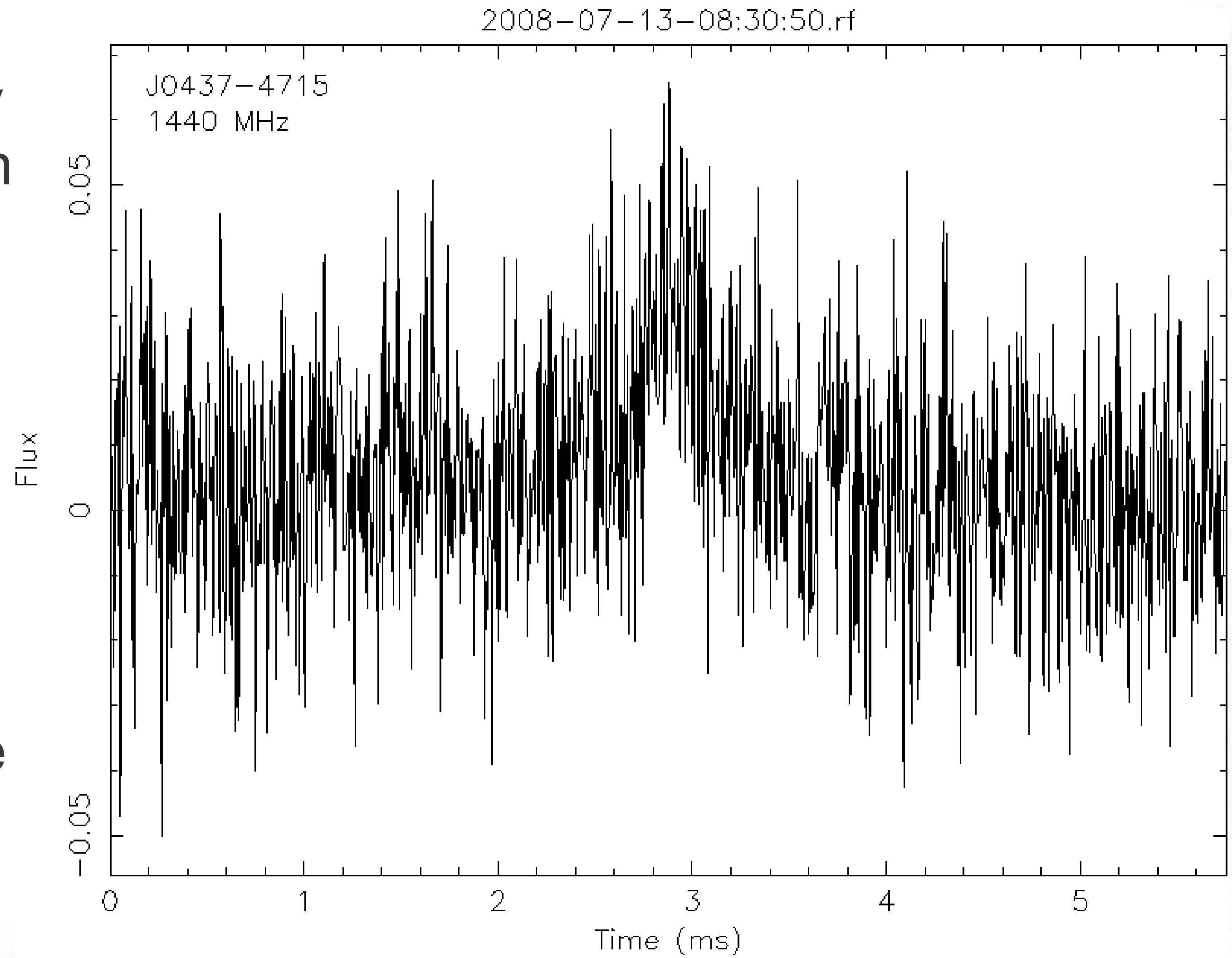
- Radio pulses from a pulsar are *almost* periodic.
  - Typical spin-down rates are $10^{-15}$ s/s.
  - As the energy loss rate is almost constant, we can treat pulsars as clocks if we correct for their steady spin-down.
- Pulsars also suffer from many systematic drifts.
  - This is both a benefit and a disadvantage:
  - Measuring predictable drifts allows us to determine the properties of the pulsar system.
  - However, it complicates their use as clocks.

# Measuring Arrival Times

- The usefulness of an oscillator depends on the precision to which we can measure each “tick”.
  - Less of a problem on Earth where we can build cavity resonators, amplifiers, etc.
  - For pulsars, we only get a weak radio signal.
  - Timing precision can be of order 100 ns for the best pulsars, but most are not suitable for high precision timing.
- Pulsars can only be used as clocks if we understand (can *predict*) all systematic drifts.

# Timing a Pulse

- How could we most precisely measure when this pulse arrived?
- What factors contribute to the level of precision?
- Which pulsars are likely to be the best for timing?



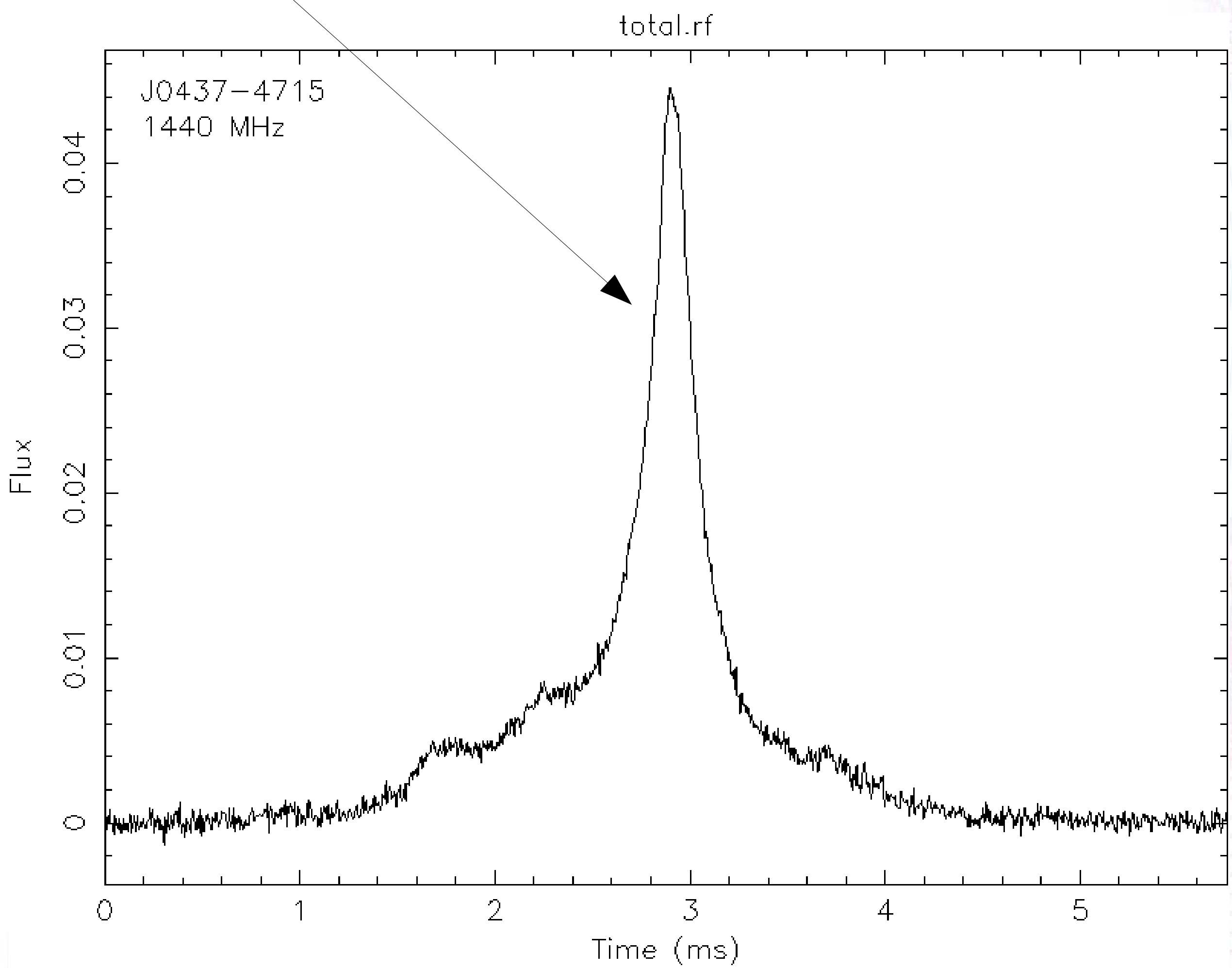
# Pulse Stability

- Single pulses from a pulsar tend to have different shapes, strengths, phase jitter, etc.
- It was discovered that the pulse “profile” usually stabilises after ~10 000 pulses are added.
- Timing experiments are done on “integrated” profiles, not single pulses.
- The arrival “time” measured is usually taken to match that of a pulse arriving in the middle of the averaged span.

# Matched Filtering

- Assume some knowledge of the “intrinsic” pulse profile.
- Correlate with the observed profile and find the peak.
- Precision can be as good as 10% of the “bin” width.
- Fourier domain correlation gives noise-limited precision.

Standard template profile, constructed from all available data.



# Improving Timing Precision

- Maximise the S/N of observations.
  - Observe bright pulsars.
  - Maximise the recorded radio bandwidth.
  - Use more sensitive telescopes.
  - Correct for dispersion smearing.
- Observe the “right kind” of pulsars.
  - Those with rapid spin periods.
  - Those with the most stable spin periods.
  - Those with sharp edges in their pulse profile.

# Quantifying Timing Precision

- The radiometer equation for a pulsed source includes contributions from two noise levels (on and off pulse) and has the following form:

$$S/N = \sqrt{n_p t_{\mathrm{obs}} \Delta f} \left( \frac{T_{\mathrm{pulse}}}{T_{\mathrm{sys}}} \right) \frac{\sqrt{W(P-W)}}{P}$$

Width
Period

- Uncertainties in Time Of Arrival (TOA) estimation go as the pulse width over S/N:

$$\sigma_{\mathrm{TOA}} \simeq \frac{W}{S/N} \propto \frac{S_{\mathrm{sys}}}{\sqrt{n_p t_{\mathrm{obs}} \Delta f}} \times \frac{P\,\delta^{3/2}}{S_{\mathrm{pulsar}}} \qquad \delta = \frac{W}{P}$$

# Timing Precision Exercise

- *Which pulsar is better for timing if we observe with the same telescope configuration?*
- a) J1909-3744 with a period of 2.9 ms, pulse width of 43 µs and mean flux of 3 mJy.
- b) J0835-4510 with a period of 89 ms, pulse width of 2.1 ms and mean flux of 1.1 Jy.

$$\sigma_{\mathrm{TOA}} \simeq \frac{W}{S/N} \propto \frac{S_{\mathrm{sys}}}{\sqrt{n_p t_{\mathrm{obs}} \Delta f}} \times \frac{P \delta^{3/2}}{S_{\mathrm{pulsar}}}$$

# Millisecond Pulsars

- “Recycled” pulsars, spun up by matter accretion from a binary companion.
- Accretion process reduces magnetic field strength and hence spin-down torque.
  - Boosts rotational stability.
- Transfer of angular momentum from accreting matter also makes the pulsar spin faster.
- For these reasons, MSPs are the best for high precision timing experiments.

# Time Tagging (1)

- There is not an established pulsar-based time standard, though pulsars might be better than atomic clocks on the longest time scales.
- Pulse arrival times are measured against existing global time standards (UTC).
- Observatories need a good *local* time standard.
  - Usually a Hydrogen maser, stable over short periods but tends to drift in the long term.
- This is tracked and compared to global atomic time using GPS satellites.

# Time Tagging (2)

- Pulsar data archives store the precise time of the first recorded sample according to the telescope's local time standard.
- This can be converted to UTC by applying *retrospective* clock corrections.
- The measured “shift” of the observed pulse is added to the starting time to give the “topocentric” arrival time.
  - This is the arrival time as measured at the observatory, using a global time standard.

# Timing Models (1)

- As we will see, it is useful to construct a mathematical model of the rotation of a pulsar.
- The first stage is to express the spin frequency as a Taylor series expansion:

$$\nu(t) = \nu_0 + \dot{\nu}_0 (t - t_0) + \frac{1}{2} \ddot{\nu}_0 (t - t_0)^2 + \ldots$$

- We can directly measure the spin frequency and its first derivative at a reference time.
  - Sometimes, young pulsars have a measurable spin second derivative, but this term can also be included to absorb "timing noise".

# Timing Perturbations (1)

- Consider a solitary pulsar. After removing the spin-down effect described on the previous slide, what other systematic trends might exist in the pulse arrival times?
  - Hint: Think of the Doppler effect and relativity.

# Transforming to the Barycentre

- To remove all systematic trends due to the motion of the Earth, we need to find an inertial reference frame.
  - The closest inertial frame is that associated with the centre of mass of the Solar system (SSB).
- We need to calculate the time an observed pulse *would have* arrived at the SSB:

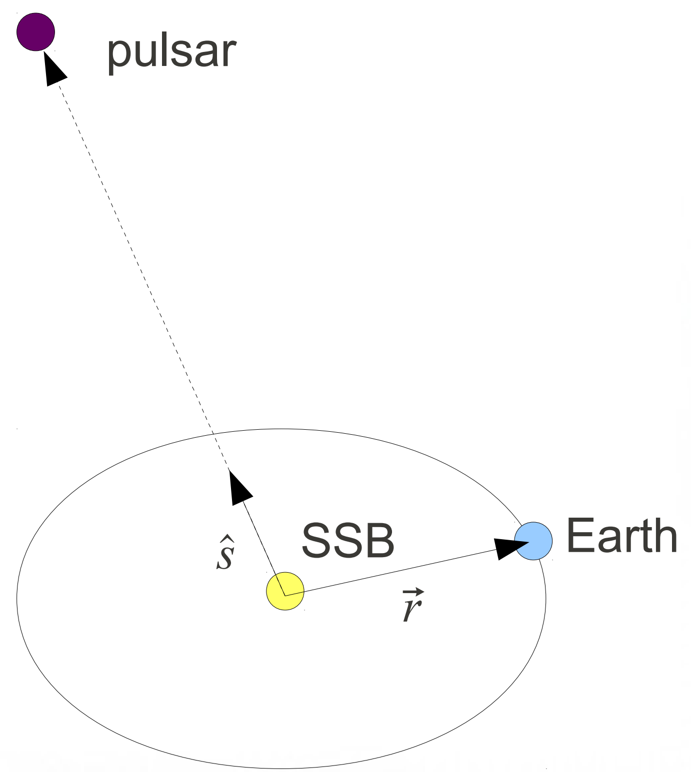
$$t_{\mathrm{SSB}} = t_{\mathrm{topo}} - \Delta D / f^2 + \Delta_{\mathrm{R}} + \Delta_{\mathrm{S}} + \Delta_{\mathrm{E}}$$

Infinite frequency correction: $\dfrac{e^2 DM}{2\pi m_e c} \approx DM \times 4149\,\mathrm{MHz}^2\,\mathrm{pc}^{-1}\,\mathrm{cm}^3\,\mathrm{s}$

# Römer Delay

- The time it takes light to travel from the telescope to the SSB.
- Classical, geometric path delay effect.
- In practice, we first correct the observatory TOA to the geocentre, then to the SSB.

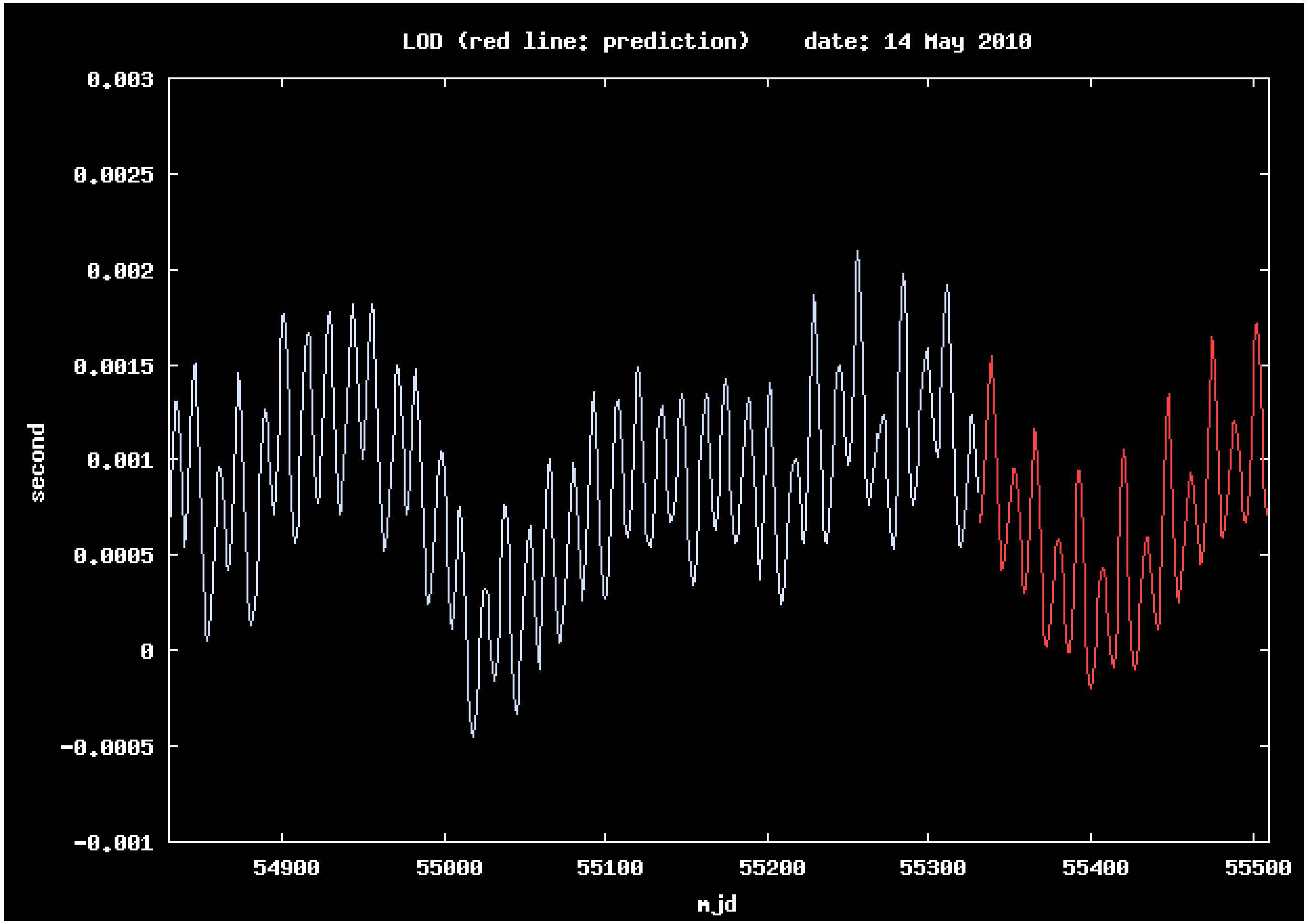
$$\Delta_R = -\frac{1}{c}\vec{r}\cdot\hat{s}$$



# Solar System Ephemerides

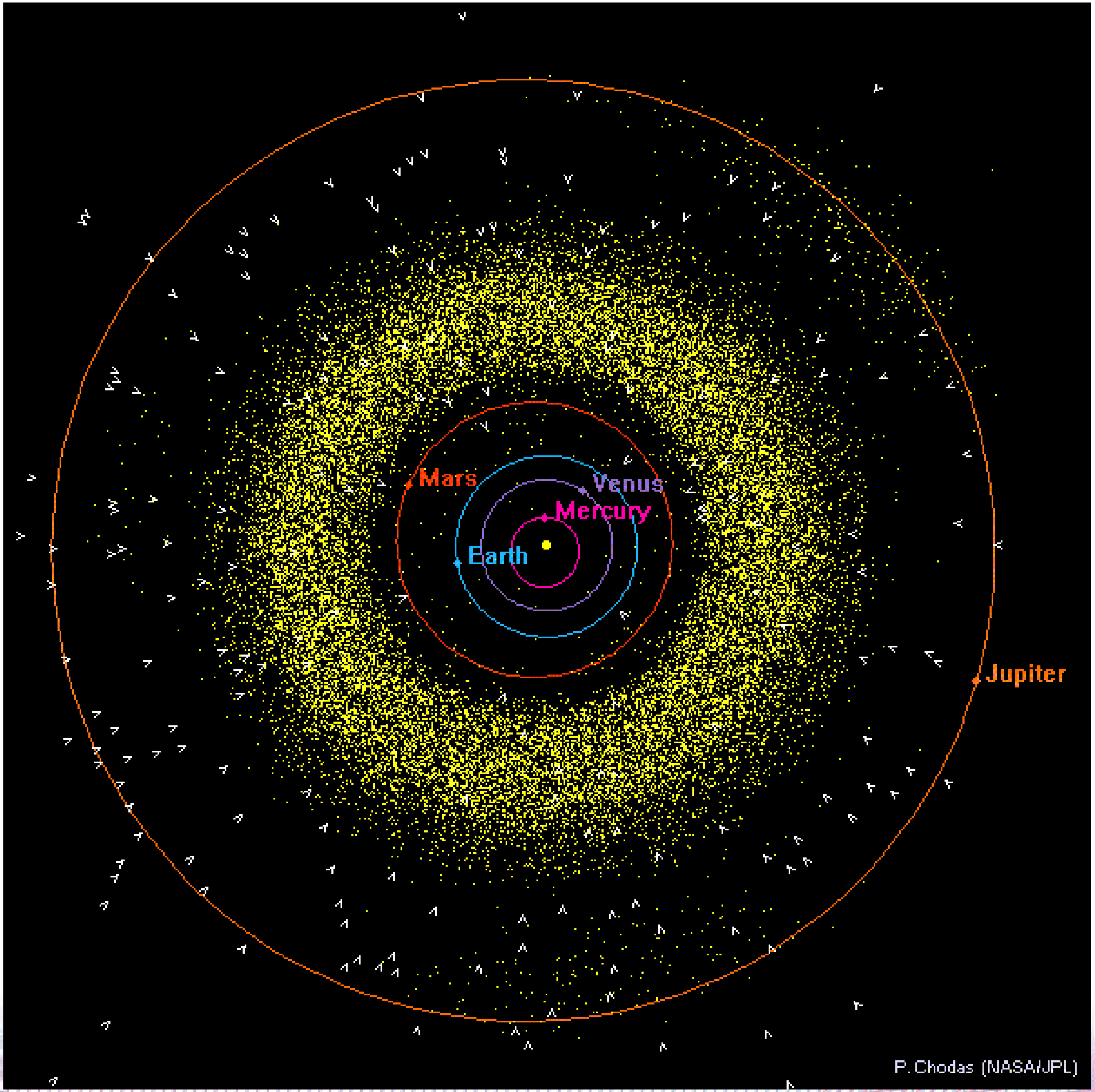
- To compute the vectors on the previous slide, we need to know two things:
  - How the Earth rotates and exactly where on its surface our telescope is.
  - Where all the significantly massive bodies in the Solar system are.
- Earth rotation is monitored by the IERS.
  - See http://www.iers.org
- Planetary positions are monitored by JPL.
  - See http://ssd.jpl.nasa.gov

# Length of Day

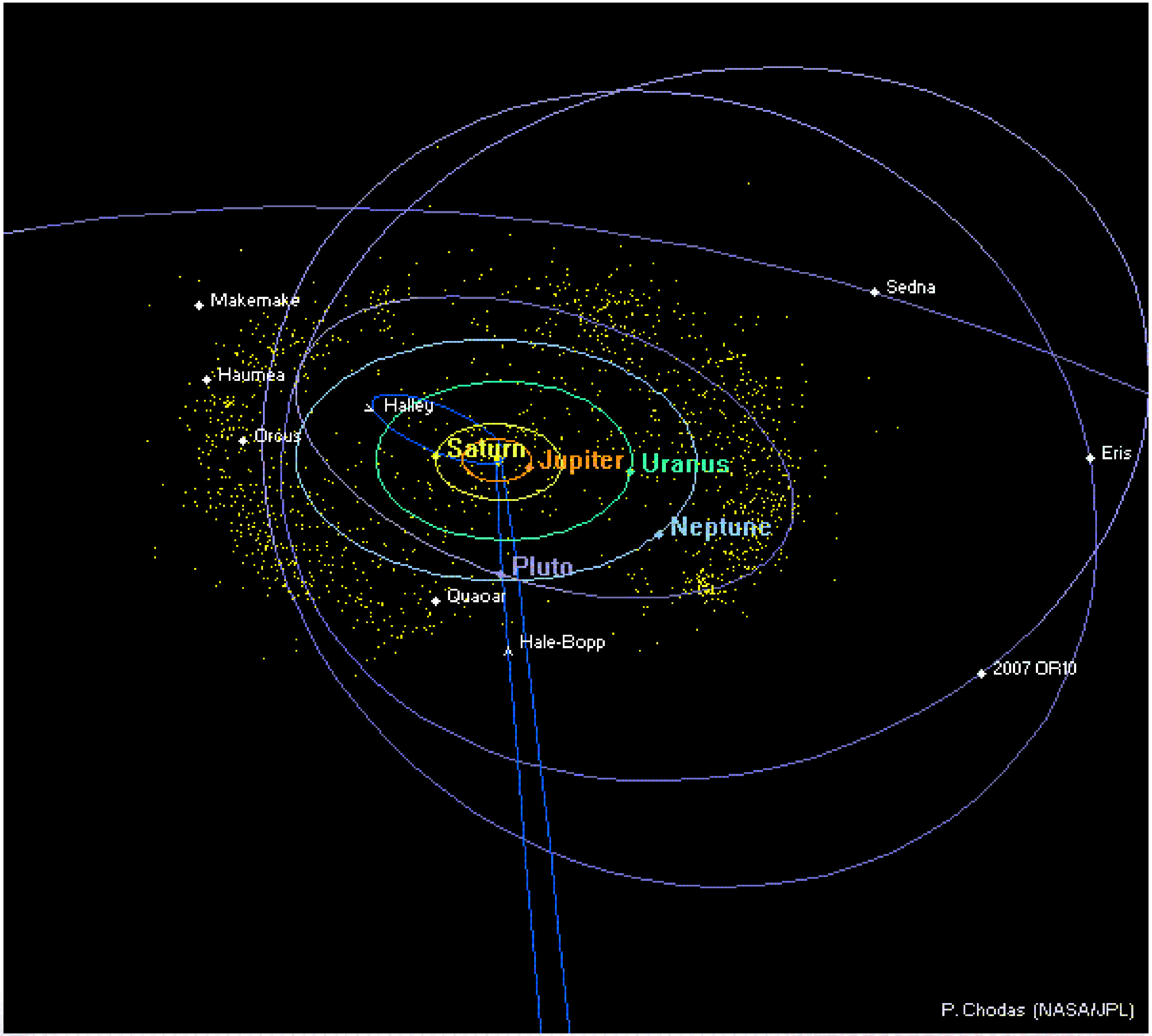



Aoki,S., Guinot, B., Kaplan, G.H., Kinoshita, H., McCarthy, D.D., Seidelmann, P.K. 1982: Astron. Astrophys. 105, 1.

Inner Solar system, April 1st, 2010.



Outer Solar system, April 1st, 2010.



# The Pulsar's Position

- Our knowledge of $\hat{s}$ depends on the position of the pulsar as well as the SSB.
- With no delay correction, the pulse arrival times will be in error by up to:

$$\Delta_R^{\max} = \frac{1\,\mathrm{AU}}{c} \cos\beta \approx 500 \cos\beta \quad \text{seconds}$$

- Where β is the ecliptic latitude of the pulsar.
- Timing allows a precise position measurement, unless the pulsar is near the ecliptic plane.

# Proper Motion (1)

- Pulsars can move with respect to the SSB.
- This introduces a time dependence to the location of the source on the sky.
- Pulsar timing is sensitive to the transverse component of this motion as it introduces a changing Romer delay.
  - To first order, a Doppler shift introduced by radial motion is absorbed into the determination of the pulsar's spin period.

# Proper Motion (2)

- In equatorial coordinates (α = RA, δ = Dec):

$$\mu_\delta \equiv \dot{\delta}\,,\; \mu_\alpha \equiv \dot{\alpha}\cos\delta\,,\; \mu_\mathrm{T} = \sqrt{\mu_\alpha^2 + \mu_\delta^2}$$

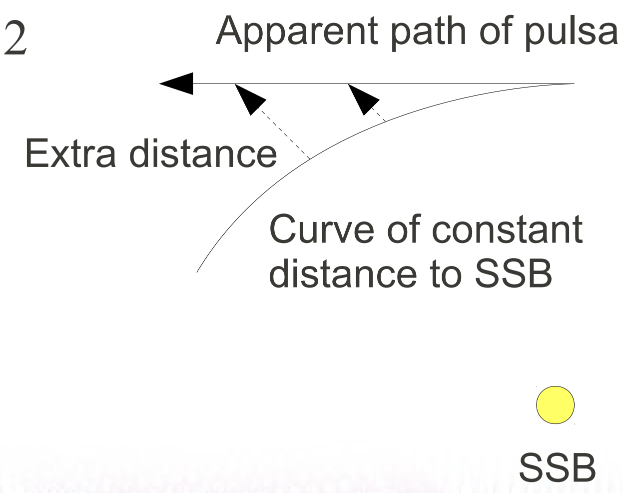
- If the distance to the pulsar is known, this corresponds to a transverse velocity:

$$V_\mathrm{T} = 4.74\,\mathrm{km\,s^{-1}} \left( \frac{\mu_\mathrm{T}}{\mathrm{mas\;yr^{-1}}} \right) \left( \frac{d}{\mathrm{kpc}} \right)$$

# The Shklovskii Effect

- Transverse motion leads to a secular increase in the distance from the pulsar to the SSB.
  - This mimics an acceleration as the pulses arrive from an ever-increasing distance.
  - The extra delay is a quadratic function of time:

$$\Delta t_S = \frac{V_T^2}{2\mathrm{dc}} t^2$$



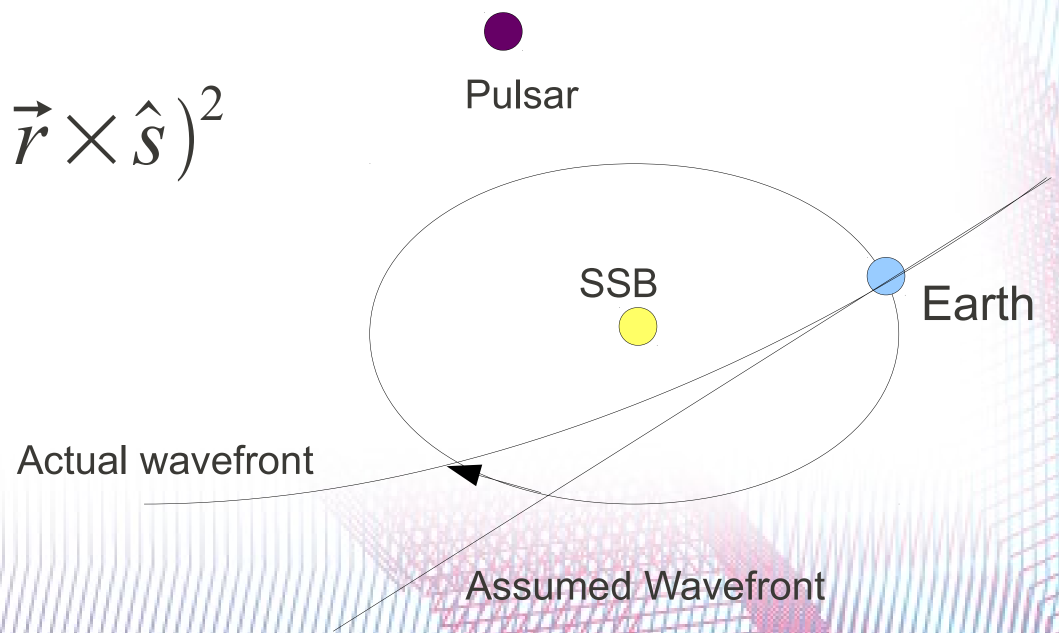

- This introduces an apparent period derivative:

$$\frac{\dot{P}}{P} = \frac{1}{c}\frac{V_\mathrm{T}^2}{d}$$

# Timing Parallax

- Although pulsars are unresolved point sources in the beam of a single-dish telescope, they can exhibit a kind of parallax.
- Timing parallax comes about because the delay due to wavefront curvature across Earth's orbit can be significant.
- Delay: $\Delta t_{\pi} = -\frac{1}{2cd}(\vec{r} \times \hat{s})^2$
- Amplitude:

$$\frac{(1\,\mathrm{AU})^2 \cos\beta}{2cd}$$



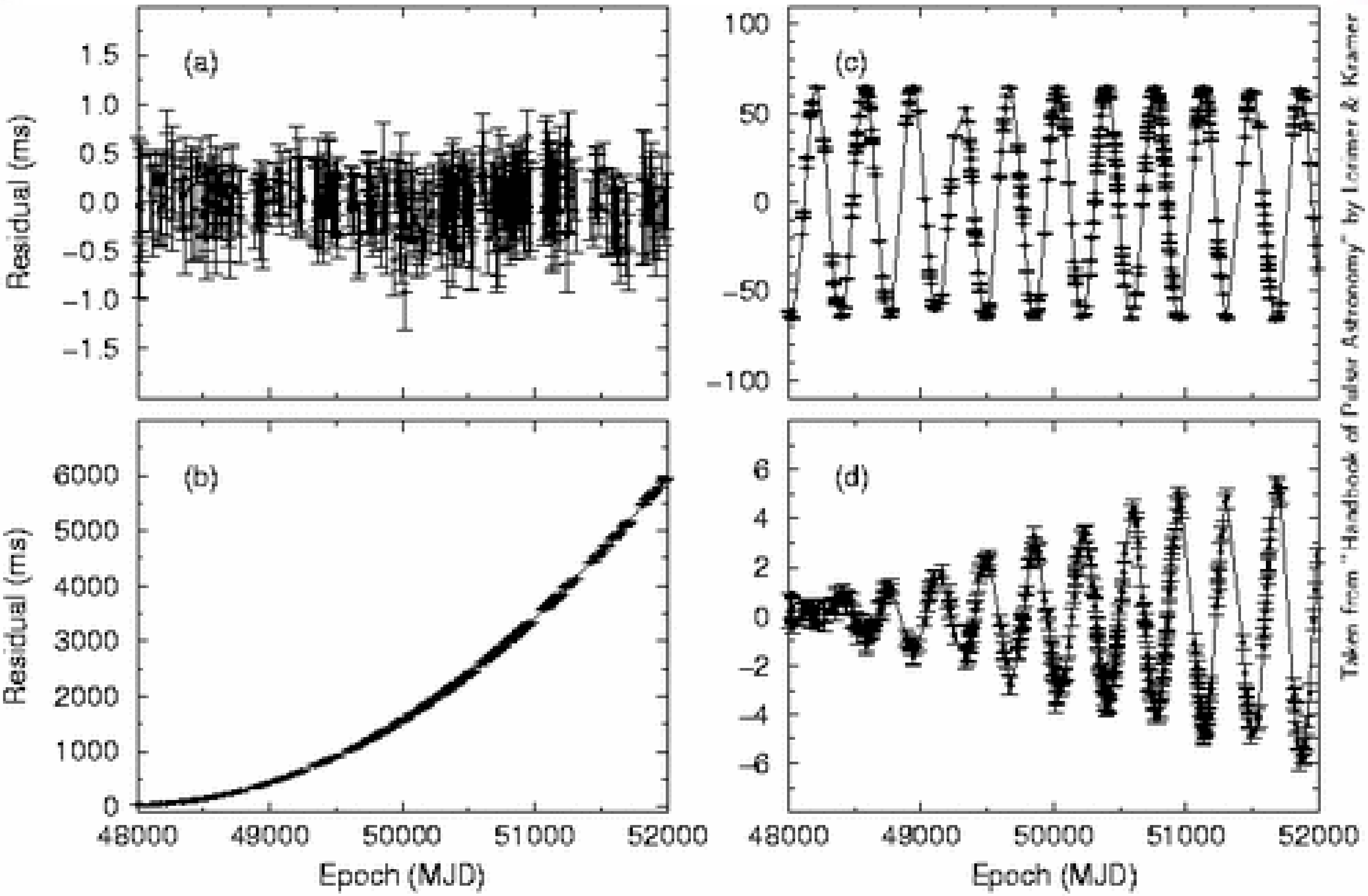
# Relativistic Corrections

- $\Delta_S$ is the Shapiro delay caused by the passage of light through curved space-time in the vicinity of massive objects in the Solar system.
  - Only important if the line of sight to the pulsar grazes the Sun's limb or Jupiter's position.
- $\Delta_E$ is the Einstein delay, caused by time dilation due to the motion of the Earth and other effects.
- These are “higher order” corrections to the timing model, but still significant.

# Timing Residuals

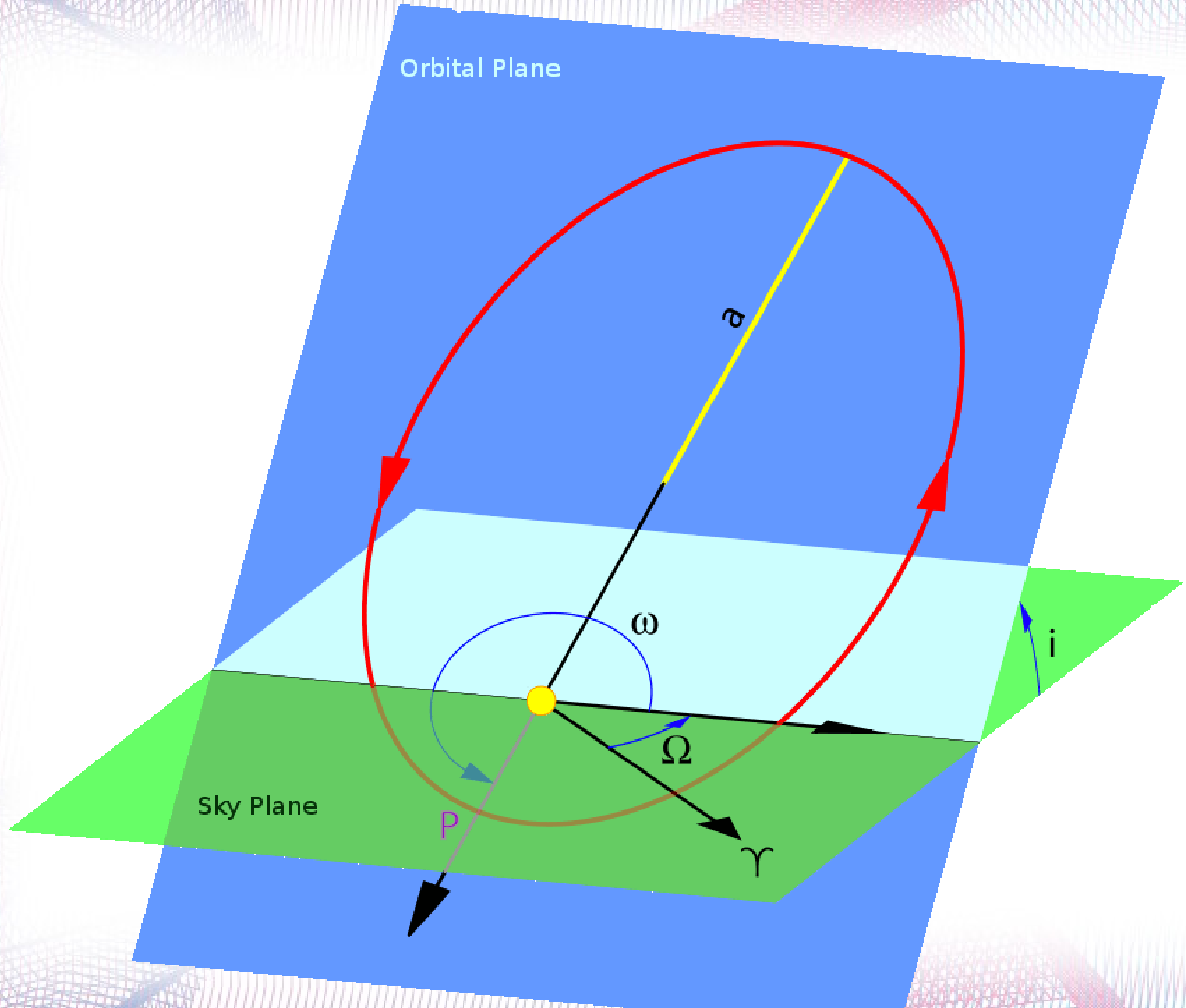
- As we have seen, many different perturbations act on the observed spin period of a pulsar.
- To cope with this we construct a timing model that sums the contributions from all known perturbations.
- To refine the model we minimise the least-squares difference between the predicted and observed pulse arrival times (residuals).
- A perfect model should yield *white noise timing residuals*.

# Timing Signatures



# Keplerian Orbits

- About 1% of the known pulsars have binary companions. In this case, additional parameters must be included in the timing model:
  - Orbital Period, $P_b$
  - Projected semi-major orbital axis, $x = a \sin(i)$
  - Orbital eccentricity, $e$
  - Longitude of periastron, $\omega$
  - Epoch of periastron passage, $T_0$
  - Position angle of ascending node, $\Omega$



Orbital Plane
a
ω
i
Ω
Sky Plane
P
♈

# The Mass Function

- A binary pulsar is equivalent to a spectroscopic binary system where only one star is bright enough to detect.
- In this case, Kepler's 3rd law gives:

$$\frac{(m_c \sin i)^3}{(m_p + m_c)^2} = \frac{4\pi^2}{G} \frac{(a_p \sin i)^3}{P_b^2}$$

- This quantity is called the “mass function”.
  - While everything on the RHS is observable, the masses and inclination are unknown and cannot be determined in a Keplerian system.

# Post-Keplerian Parameters (1)

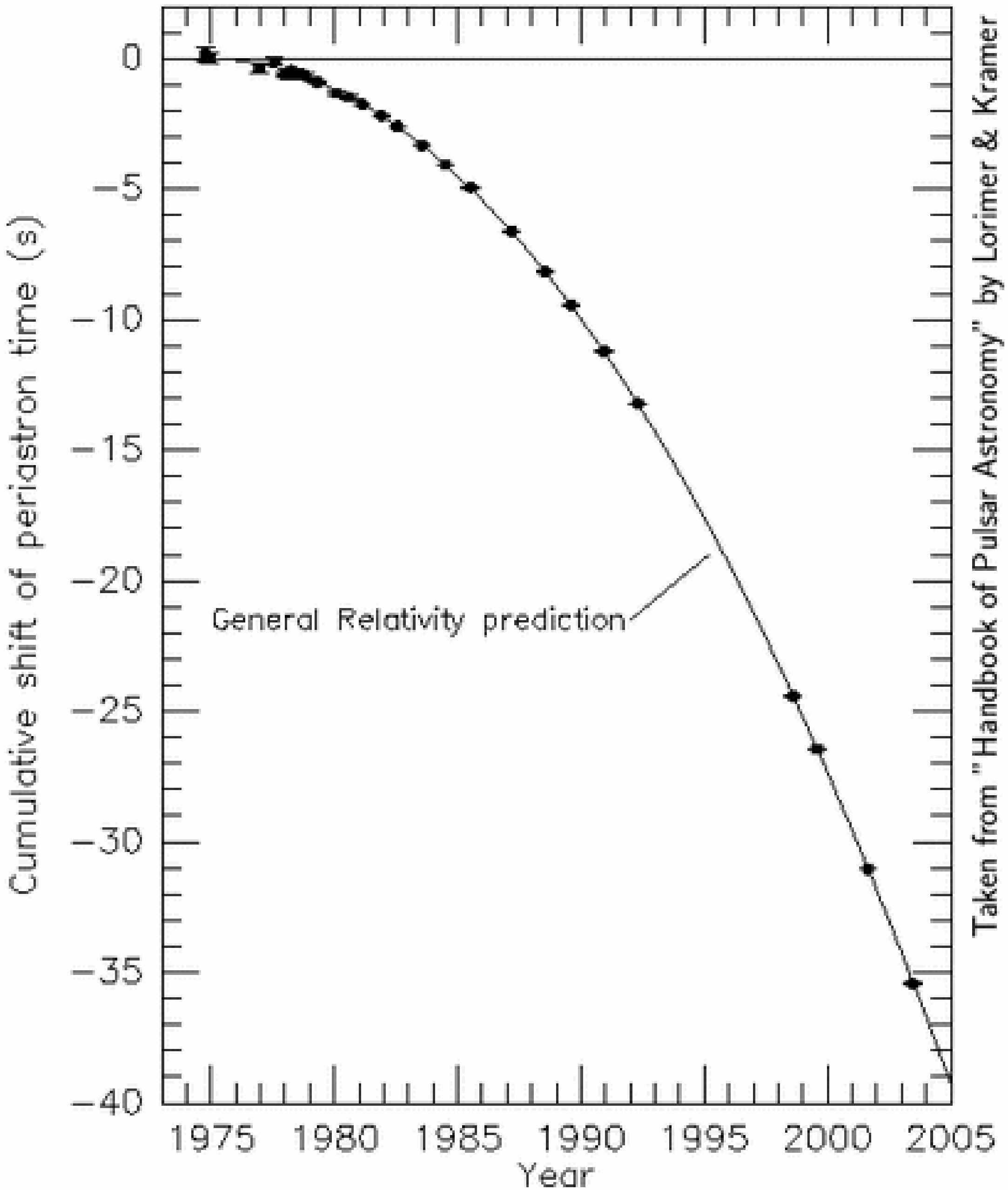
- Neutron stars are compact objects and can exist in tight binary systems.
  - High orbital velocities, strong gravitational fields and rapid accelerations.
- Post-Keplerian corrections to gravitational theories become important.
- Pulsars and white dwarf stars can be considered point masses. In GR, we obtain another 5 timing model parameters.
  - These are functions of the component masses and other Keplerian parameters.

# Post-Keplerian Parameters (2)

- Relativistic effects include:
  - *Precession of periastron*. The precession of Mercury's orbit was used as an early test of GR. The relativistic component has a magnitude of only 43” per century, while double neutron stars can exhibit several degrees of precession per year.
  - *Orbital period decay*. The emission of quadrupole gravitational radiation bleeds energy from the orbit, causing it to shrink with time. Radiated power is higher in compact systems with massive components.

# The Original Binary Pulsar

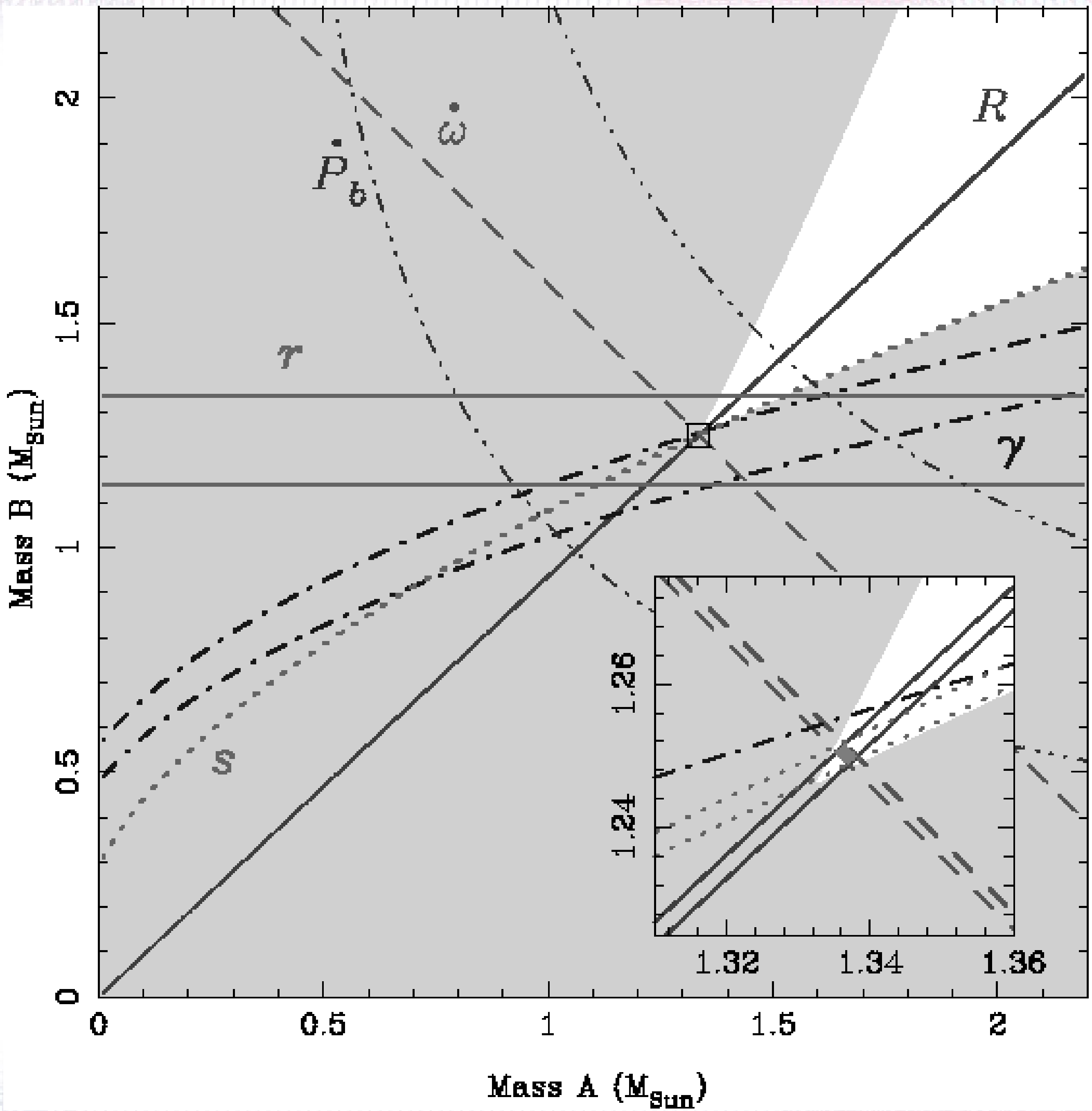
- A double neutron star system in which one is an observable radio pulsar.
- Discovered in 1974 by Hulse & Taylor, who received the Nobel Prize in 1993.
- Two compact, 1.4 Solar mass objects, with an orbital period of 7.75 hours!
- Orbital period has been observed to decrease at a rate of $2.421 \times 10^{-12}$ s/s (within uncertainty!).
- Orbit shrinks by 1cm per day.
  - Eventually, the neutron stars will merge!



Cumulative shift of periastron time (s)
0
−5
−10
−15
−20
−25
−30
−35
−40
1975
1980
1985
1990
1995
2000
2005
Year
General Relativity prediction
Taken from "Handbook of Pulsar Astronomy" by Lorimer & Kramer

# The Double Pulsar

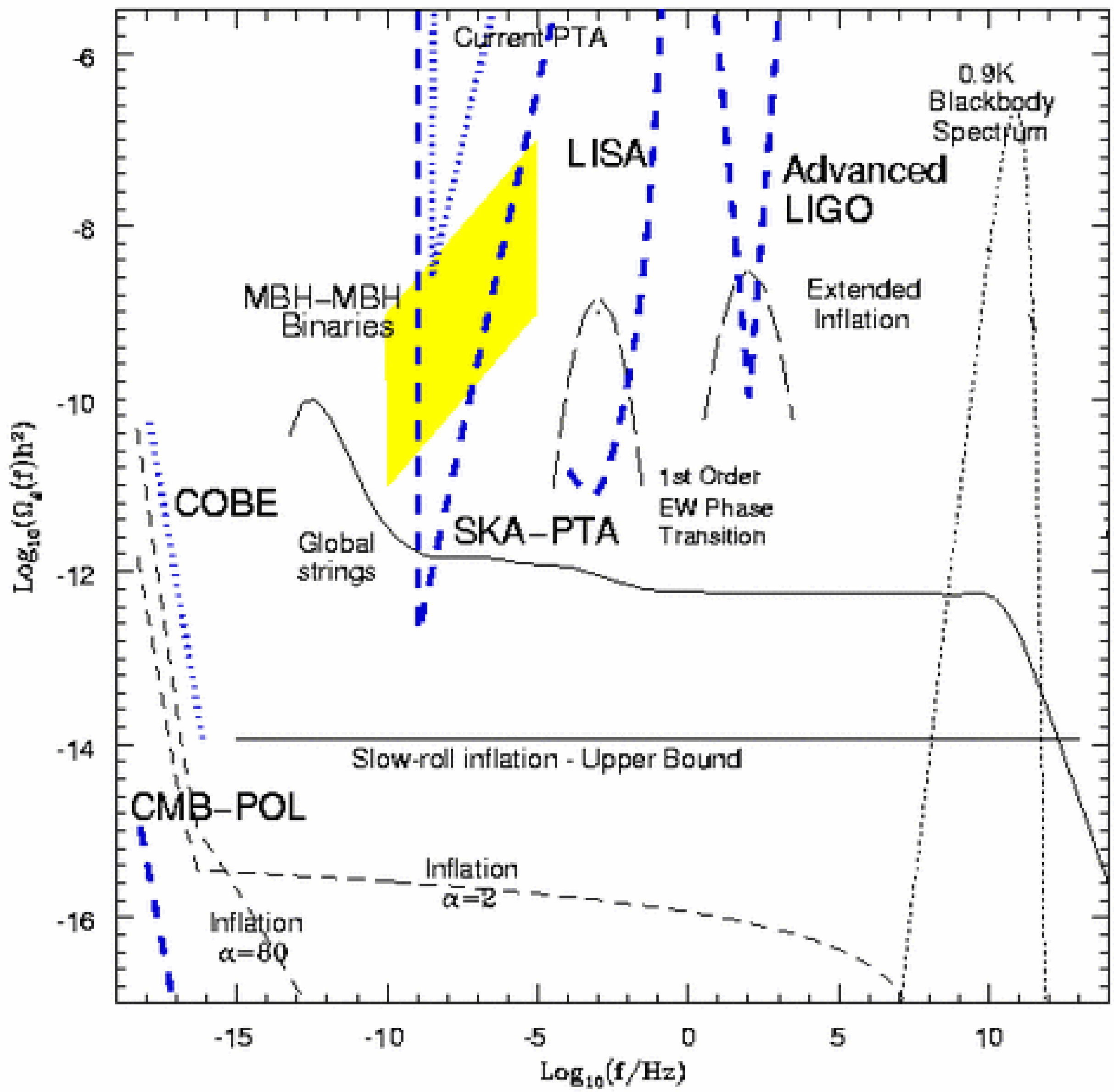
- Only one known example of a binary system in which both components are visible pulsars.
- Discovered in 2003/4 (not simultaneously!)
- Very compact, orbital period of 2.4 hours.
- Timing of each pulsar gives an independent mass function solution and the ratio of the semi-major axes gives the component mass ratio (as with a visual binary system).
- Measuring just one PK parameter uniquely determines both neutron star masses.




Taken from "Handbook of Pulsar Astronomy" by Lorimer & Kramer

# Pulsar Timing Arrays

- Timing of single and double pulsars is very informative, but can we time multiple pulsars?
  - We might be able to detect systematic trends that are coherent across the whole Galaxy.
  - The *gravitational wave background* signal.
- Models predict that we need ~20 pulsars timed to a precision of ~100 ns for ~5 years.
- This could be a unique probe for the low-frequency end of the gravity wave spectrum.

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