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Piston Airplane Cruise Performance

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PISTON AIRPLANE CRUISE PERFORMANCE

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Ability to achieve efficient range and endurance performance can mean the difference between an uneventful flight and one which ends in anxiety or even tragedy. Beyond the economics of fuel costs, the presence of unexpectedly strong headwinds, navigational error, or deteriorating weather may test the pilot's cruise management capability. The prudent pilot will be prepared by thoroughly understanding the principles underlying cruise performance.

Federal Aviation Regulations Part 61 requires Commercial Pilot applicants to have received instruction in maximum performance takeoffs, landings, climbs, and descents. Conspicuously absent is any requirement for instruction in maximum performance cruise, where the vast majority of flight actually occurs. Although the Commercial Pilot requires 50 hours of cross-country flights, there is no requirement that understanding of the principles involved be achieved.

The *Flight Training Handbook* (1980) devotes three pages to the effects of variables, but provides no practical guidance. Advanced performance texts employ calculus techniques to derive theoretical results of little practical use to pilots. No questions or instruction on optimum cruise planning are found in Commercial Pilot study guides. In summary, the Commercial Pilot is neither required nor encouraged to gain practical competence in efficient cruise planning and management.

Planning and executing efficient cruise profiles require logical integration of five variables. These are power, altitude, speed, weight, and wind. Whether the objective is saving time, fuel, or both, interdependence among the variables must be appreciated. Although the subject is complex, it can be approached logically. First, theory will be explored, then several representative airplane examples used to test the theory and examine the many tradeoffs. Procedures to minimize the adverse effects of headwinds will be presented.

The following procedures provide logical alternatives which enhance safety and operating economy. The goal is a set of cruise optimization steps which can be applied before and during flight. Although a substantial level of detail is provided, it is not necessary to follow every theoretical and mathematical detail in order to apply the fundamental concepts. Aviation educators and flight instructors are the keys to propagating the required knowledge to the piston-pilot population.

SYNOPSIS OF CONTENTS

1. Optimum calibrated airspeeds (CAS) for both maximum range and maximum endurance vary with weight, but each is conducted at a specific angle of attack (AOA) independent of weight. At constant AOA, optimum speeds are proportional to the square root of weight. Therefore, maintaining efficient range or endurance flight requires progressive power and speed reductions as fuel is burned.

2. Maximum endurance (time aloft) corresponds to minimum fuel flow (FF) and engine power output required to maintain altitude. The power required for maximum endurance flight is very low, typically about 30% of rated power. For endurance, the lower the altitude the better.

3. a. Neglecting wind effects and fuel burned during climb and descent to and from cruising altitude, available maximum range is independent of altitude.

b. Maximum range CAS and AOA are constant for a given weight, independent of altitude. However, true airspeed (TAS) and power required increase with altitude as density ratio (actual density compared to standard sea level density) decreases. The ratio between TAS and CAS is the reciprocal of the square root of the density ratio. This ratio is termed "SMOE" (which derives from standard means of evaluation). **Table 4** contains SMOE versus density altitude in abbreviated form, but in practice SMOE normally is calculated using an analog or digital flight computer. A useful thumbrule is that (for constant CAS) SMOE and TAS increase approximately 1.5% per 1,000 feet.

c. Maximum range TAS, FF, and power required all increase with altitude in direct proportion to SMOE. The key conclusion is that maximum available specific range (miles per gallon or pound of fuel) is independent of altitude.

d. The common, but mistaken, belief that piston airplane maximum range improves with altitude is based either on constant power or constant TAS, neither of which provides maximum-range flight conditions.

4. Tradeoffs between speed and range (for constant weight and altitude) are linked by complex but generic relationships best interpreted graphically. **See Figures 4 and 5.** Moderate speed increases are possible with minimum range sacrifice. Consistent with jet transport practice, the "long-range cruise" condition is defined as that speed above maximum-range speed which corresponds to a 1% range sacrifice. Piston airplanes can fly 7% above maximum-range speed and achieve 99% of their absolute maximum range.

5. Theory was compared with performance data for nine representative airplane models, as derived from their pilot operating handbook (POH) data. Deviations from theoretical performance relationships were minor and plausible.

6. In the presence of significant headwind or tailwind components, the optimum (no wind) maximum-range airspeed requires adjustment. Based on empirical data, simple and practical headwind/tailwind rules of thumb were developed.

7. Analysis of a particular airplane's cruise performance is keyed to the determination of its maximum range CAS (at standard weight). Unfortunately, this speed will not be found (explicitly) in the POH. However, four methods for estimating an airplane's maximum-range CAS (and IAS) are offered. These are:

a. listings of 10 models' characteristics **(Table 2),**

b. Kershner's rules of thumb (1985),

c. a method derived from POH performance data, and

d. a method based on a quick, simple flight test.

8. Detailed flight-planning steps are provided for two common, baseline mission profiles. These are:

a. maximum practical speed/minimum time enroute, and

b. long-range cruise (99% of absolute maximum range).

FUNDAMENTALS OF CRUISE THEORY

In analyzing piston airplane level cruise performance, one should recall that engine power output (power required, P_R) may be considered closely proportional to fuel flow (FF), and vice versa. The constant of proportionality involves brake specific fuel consumption (BSFC) and propeller efficiency, and minor limitations to this approximation will be examined later. Since FF varies with P_R, the latter can be used in lieu of FF. Other relevant fundamentals are:

1. For constant AOA flight, TAS increases with altitude in proportion to the reciprocal of the square root of the density ratio, which represents the ratio between true and calibrated airspeeds (TAS/CAS). Table 4 tabulates TAS/CAS, or SMOE.

2. Maximum endurance for any airplane is obtained at the condition of minimum FF for level flight. Minimum FF corresponds to minimum required engine power output.

3. The slope of a line from the origin to any point on the power required versus TAS (V) curve is inversely related to specific range (SR), or NM per pound of fuel. **Note Figures 1 and 2.** The shallower the slope, the greater the (V/FF) ratio and the better the SR. The tangent to the P_R curve (corresponding to the $(L/D)_{MAX}$ condition) then determines the optimum range condition, since that slope is the shallowest. Therefore piston-powered airplanes are expected to obtain maximum range at the condition of best aerodynamic efficiency or minimum drag, where the ratio of lift to drag is greatest, $(L/D)_{MAX}$.

CONSTANT ALTITUDE CRUISE

Figure 1 illustrates the effect of weight variation on

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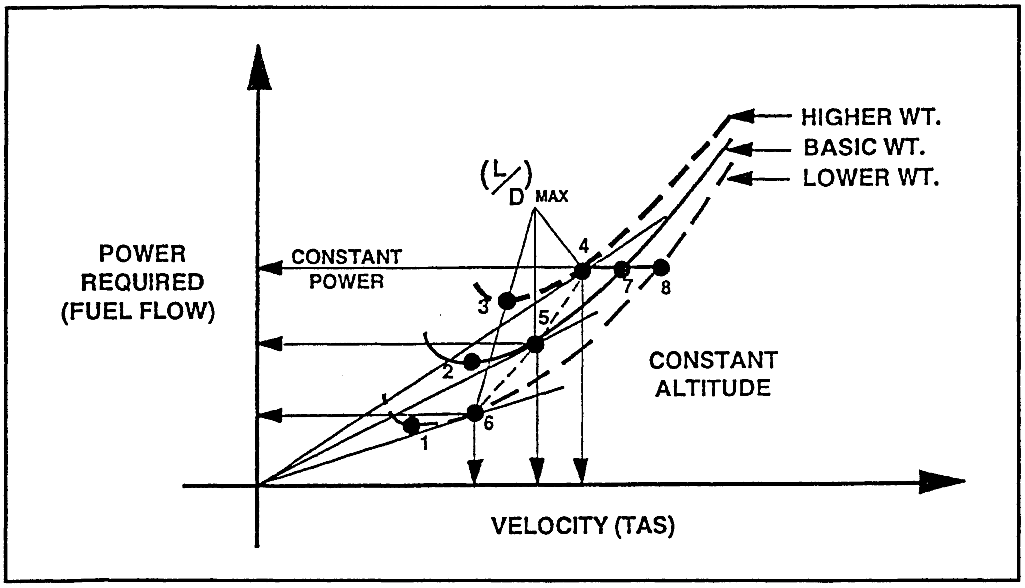
Figure 1
Effect of Gross Weight



piston airplane performance at constant altitude. Variations in weight are exaggerated for emphasis. Low points on the respective curves (1, 2, and 3) correspond to minimum P_R, hence minimum fuel flow and maximum endurance, or time aloft.

As fuel is burned and weight reduced, the corresponding V, P_R, and FF is decreased, although the AOA remains constant. No one will be surprised to learn that lighter airplanes require less power in the holding pattern.

Management of maximum-range cruise throughout a substantial weight reduction requires attention in order to match speed and power settings to variable weight. Remember the Voyager's epic non-stop flight around the world in 1986? After nine days aloft, landing weight was approximately 28% of the takeoff weight, and little useable fuel remained. Had the crew not applied correct cruise optimization principles, that flight could not have succeeded.

Assume that the heavy airplane in Figure 1 begins cruise at $(L/D)_{MAX}$ conditions, tangent to the P_R versus V curve at point 4. Subsequent cruise control can be either smart (maximum-range conditions maintained throughout) or careless (constant power maintained throughout), as in **Table 1.** The maximum-range profile follows points 4-5-6 as weight decreases. A progressive decrease in V, P_R, and FF is required to maintain the constant AOA corresponding to $(L/D)_{MAX}$. At constant AOA, speed must be reduced in proportion to the square root of weight. P_R will decrease three times faster than speed, since it varies as weight to the (3/2) power. For example, after a 10% weight reduction, V would be approximately 5% lower and the necessary P_R 15% lower than original values.

The constant power (careless) profile is 4-7-8. This profile moves progressively away from $(L/D)_{MAX}$, since speed increases as weight decreases. The careless pilot may be on the ground refueling as the smart pilot flies by with fuel to spare.

VARIABLE ALTITUDE CRUISE

Figure 2 conveys information about the effects of altitude on piston airplane cruise performance. Failure to grasp these principles explains the prevalent misconceptions regarding altitude's influence on range and endurance performance.

Maximum endurance/minimum obtainable FF is influenced by altitude, and the contrast is clear. Comparison of the P_R at points E_0 and E_1 proves that "lower is better" for recips. It should be noted that both points share the same AOA, CAS, and IAS. Note that at altitude (E_1), P_R, FF, and TAS are all SMOE times their sea level values. At low altitudes, P_R will be a smaller value than usually found in airplane handbooks, perhaps about 30% of rated power.

The influences of altitude on efficient range performance are both more important and more subtle than in the case of endurance. Points R_0 and R_1 both

Table 1
Cruise Profiles

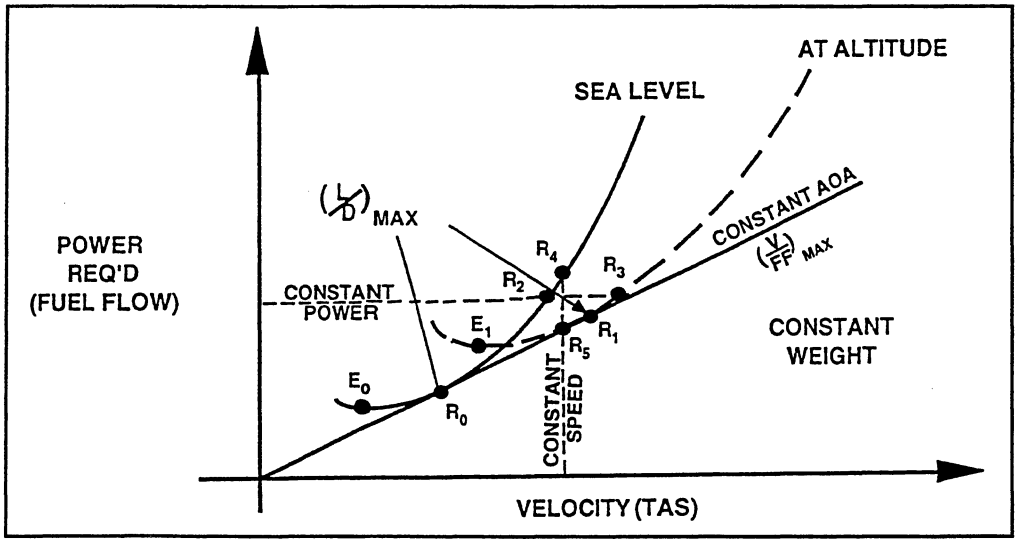
PROFILE	MEANS CONSTANT	GRADUAL SPEED
4-5-6 (SMART)	AOA, C_L, C_D, $(L/D)_{MAX}$	REDUCTION
4-7-8 (CARELESS)	POWER(%), FF	INCREASE

correspond to maximum range conditions, $(L/D)_{MAX}$, for their respective altitudes. Other speeds, either faster or slower, reduce specific range. At R_1, both speed and power are SMOE times their sea level values at R_0. It is important to note that one tangent and one slope fit all P_R versus V curves at all altitudes. Therefore, best specific range (V/FF) is identical at R_0 and R_1. Maximum range available is unaffected by altitude. Differences in actual airplanes are minor, reflecting slight variations in powerplant efficiencies.

Figure 3 represents the pilot operating handbook (POH) characteristics of a turbocharged twin (Beech A65-8200) at 7,700 pounds weight. Conformance to theory is excellent. Between sea level and 20,000 feet, maximum specific range varies only about 1% from the mean value, which is equivalent to less than a 2-knot change in the headwind or tailwind component. Peaks of the individual specific range (SR) curves occur at the same CAS, while TAS, P_R, and FF all vary in proportion to SMOE. The relatively flat peaks of the SR curves suggest that moderate speed increases are available with minimum SR penalty. This convenient characteristic can be used to advantage as will be discussed.

Unfortunately, POH curves and tables as revealing as Figure 3 are rarities. To the contrary, most handbooks obscure the effects of altitude on range performance.

Figure 2
Effect of Altitude



Similarly, misinformation is often found in other widely read sources. For example, a recent article in *Flying Magazine* (1992) asserted that:

> Efficiency — that is, miles per gallon — is strongly affected by altitude. Flying at high altitudes is always more efficient than flying lower down, simply because at a given true airspeed the indicated airspeed is lower. (p. 106)

Figure 3 shows that this common belief is incorrect. Nevertheless, instructors and other pilots often misinterpret POH data to prove their misconception that airplanes obtain better range at higher altitudes. The fallacy is that such "proof" follows the style of the POH by comparing flight either at constant (percent) power or at constant TAS, while using power/altitude combinations far from actual maximum-range conditions. In such cases the range handicap is diminished at higher altitudes where conditions are nearer optimum.

The correct interpretation can be gained from the two perspectives of Figures 2 and 3. Figure 2 represents theory while Figure 3 illustrates a POH example. The

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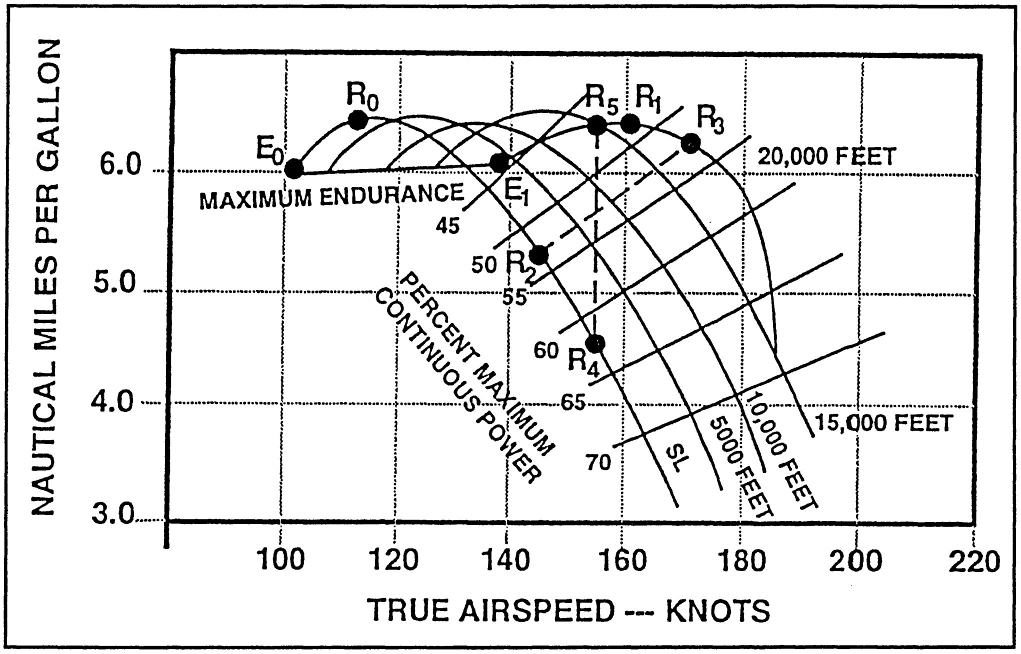
best range profile is depicted by R_0R_1. The constant power profile R_2R_3 illustrates the condition where higher altitude improves TAS and range for the same P_R and FF. Similarly, constant TAS comparisons may be made by examining profile R_4R_5. But neither constant power nor constant TAS represents maximum range profiles at varying altitudes. It is evident that relatively high power and TAS demands relatively high altitudes for cruise efficiency. For example, it should be noted in Figure 2 that both R_2 and R_4 are very remote from the sea level maximum-range condition, R_0. At altitude, both R_3 and R_5 are close to R_1, showing that the airplane is operating at a combination of power, airspeed, and altitude much nearer optimum.

Figure 3
Specific Range



VARIABLE ALTITUDE EXERCISE

Referring to Figures 2 and 3, compare maximum-range theory to conditions at R_0 and R_1, the altitude extremes. Which of the below factors are identical (I) and which are SMOE (S) times greater at R_1? Answers appear at the end of the article.

1. TAS	2. CAS	3. IAS
4. AOA	5. FF	6. Horsepower
7. C_L	8. C_D	9. L/D
10. SR (NM/lb)	11. % Power	12. No Wind Range

RANGE VERSUS SPEED TRADEOFFS

Any piston airplane's peak aerodynamic efficiency occurs at the minimum drag point (drag=D_{MIN}) corresponding to $(L/D)_{MAX}$. At this condition, parasite and induced drag are equal, with each representing half of D_{MIN}. The maximum range airspeed, V_{MR}, corresponds to D_{MIN}, and is the reference speed for discussions to follow. By use of V_{MR}, all speeds can be expressed simply as a non-dimensional ratio to this optimum airspeed, V/V_{MR}. Similarly, values for induced and parasite drag may be referenced to D_{MIN} and the actual speed ratio, V/V_{MR}. Thus, at speed V, parasite drag equals $0.5(V/V_{MR})^2$ and induced drag equals $0.5(V/V_{MR})^{-2}$.

By definition, maximum range (R_{MAX}) occurs at V_{MR}. A similar non-dimensional ratio of actual range to maximum range, R/R_{MAX}, can be determined for any V/V_{MR} ratio. Resulting equations, curves, and conclusions linking R/R_{MAX} to V/V_{MR} are therefore generic and can be related to any piston-powered airplane for which V_{MR} (CAS) is known.

The total drag at any V/V_{MR} can be expressed as:

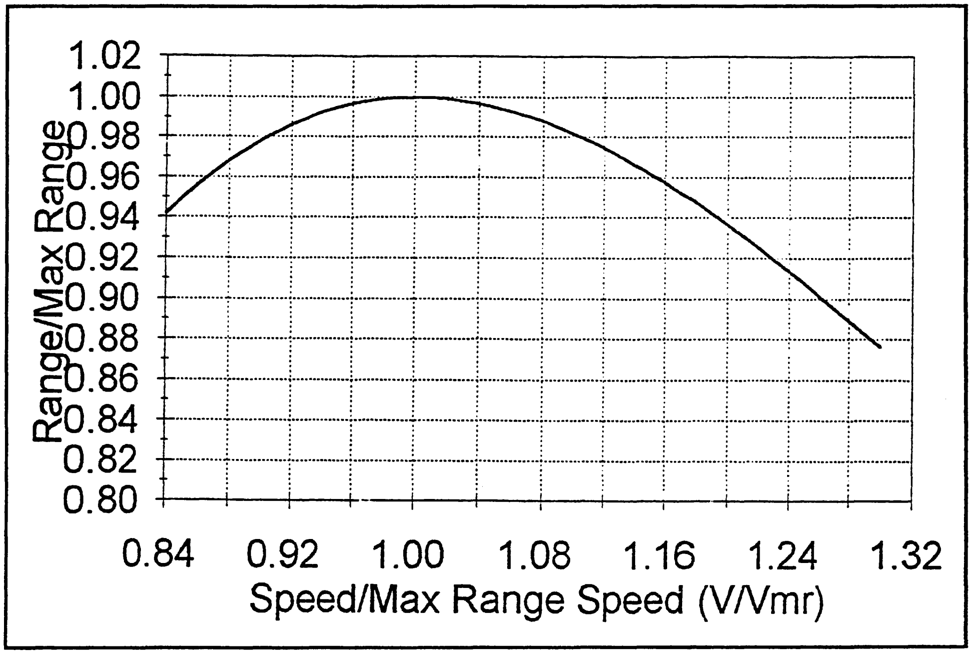
$$D=0.5D_{MIN}[(V/V_{MR})^2+(V_{MR}/V)^2] \quad \text{(Eq. 1)}$$

D_{MIN} is constant for a given weight and configuration. Since range is inversely proportional to drag, it follows that:

$$R/R_{MAX}=D_{MIN}/D=2/[(V/V_{MR})^2+(V_{MR}/V)^2] \quad \text{(Eq.2)}$$

Equation 2 has been plotted across the speed range of interest and is shown as Figure 4. Note the curve's resemblance to those of Figure 3. The relatively flat top shows that airspeeds slightly above V_{MR} cause only small

Figure 4
Range versus Airspeed



.91 and $1.20V_{MR}$. Differences between the curves are explained by re-examining the original assumption of constant propeller and engine efficiencies. In reality, the match between airframe, engine, and propeller is optimized for the regime near V_{MR} and V_{LRC}. At higher and lower speeds and power settings, powerplant efficiencies deteriorate progressively, as shown by the lower (POH-based) curve.

Surprisingly, no generic peculiarities related to powerplant type were observed. Fixed pitch, constant speed, and turbocharged samples interspersed without obvious patterns. Best fit equations for the (assumed) parabolic composite POH curve were:

For $V/V_{MR}>1$: $R/R_{MAX}=[1-1.80(V/V_{MR}-1)^2]$ (Eq. 3a)
For $V/V_{MR}<1$: $R/R_{MAX}=[1-3.33(1-V/V_{MR})^2]$ (Eq. 3b)

reductions in range. As is the standard for transport airplanes, long-range cruise speed (V_{LRC}) is defined as that speed above V_{MR} where 99% of absolute maximum range occurs. As seen in Figure 4, $V/V_{MR}=1.07$ when $R/R_{MAX}=.99$. A 7% speed increase costs only a 1% decrease in range, although speeds above $V_{LRC}=1.07V_{MR}$ are seen to decrease range rapidly.

The obvious question is how well the theory of Equation 2 and Figure 4 agrees with actual range performance of various piston airplanes. For comparison, POH data were extracted from the cruise performance data for nine different airplanes. There were three single-engine fixed gear, three single-engine retractable gear/constant-speed propeller, and three multi-engine airplanes (two turbocharged). A variety of representative altitude, airspeed, and power setting combinations were included. A total of 118 usable data points were derived, covering the speed regime between 0.87 and 1.30 times V_{MR}. A least squares curve fit was obtained for the POH data. The two are compared in Figure 5.

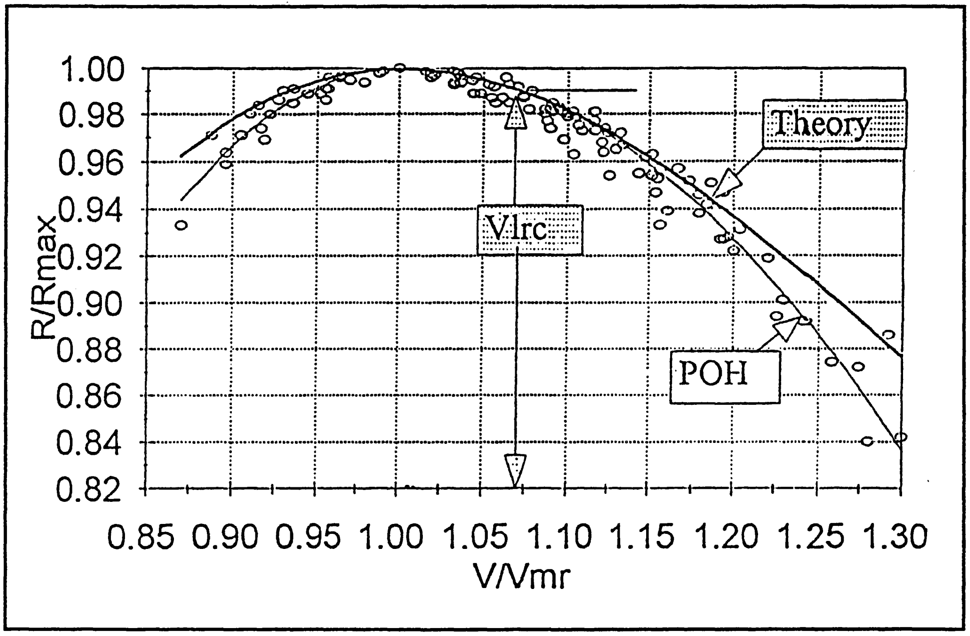
Excellent agreement between curves is seen in the regime near V_{MR}, and differences are within 1% between

Aircraft manufacturers supply pilots with various "V speeds" for normal and emergency operations, and the number of such speeds contained in handbooks averages at least a dozen per installed engine. Unfortunately, maximum range (V_{MR}) and maximum endurance (V_{ME}) speeds are not among them. Hence, 10 airplane handbooks were analyzed to estimate their maximum range and endurance airspeeds. A summary of results is contained in Table 2. All tabulated speeds are knots calibrated airspeed referenced to maximum gross weight.

HEADWIND AND TAILWIND MANAGEMENT

The existence of a headwind or tailwind respectively decreases or increases range from the no-wind conditions found in basic POH curves and tables. Whenever practical, speed adjustments should be used to optimize either condition. In particular, strong headwinds reduce range severely, especially when not managed correctly. In effect, the longer a headwind works on the airplane, the greater the influence. Conversely, prolonging a tailwind increases the advantage. Therefore, the rule is that headwinds call for increasing airspeed, and tailwinds for decreasing speed, compared to the no-wind maximum range airspeed, V_{MR}. Analysis of the POH-based empirical data curve represented in Figure 5 and Equation 3a provided the following practical rules:

Figure 5
Range versus Airspeed
(Theory and POH Data)



HEADWIND RULES OF THUMB

1. If cruising at or above V_{LRC}, do not adjust speed unless headwind component exceeds 25% of V_{LRC} (TAS). Since $V_{LRC}=1.07V_{MR}$, minor headwind conditions receive automatic compensation.

2. For each 5 knots that headwind exceeds the .25 V_{LRC} threshold, increase cruise TAS 2 knots above no-wind V_{LRC}.

3. Example: $V_{LRC}=120$ KTAS and headwind component is 60 knots. Excess headwind is 60-.25x120=30 knots. Therefore, cruise speed should be increased to 120+(6x2)=132 KTAS. Ground speed is increased from 60 to 72 knots, reducing enroute time by 17%, while fuel burned (per ground mile) is reduced 3.7%.

TAILWIND RULE OF THUMB

Decrease TAS 1 knot for every 2 knots of tailwind component, but not below $0.8V_{LRC}$. Example: $V_{LRC}=120$ KTAS and tailwind component is 36 knots. To maximize range, decrease speed to 120-.5x36=102 KTAS=$.85V_{LRC}$. Specific range increases 3.5%, although enroute time is increased 13%.

MAXIMUM ENDURANCE FLIGHT

Most POH data for endurance flight provide times corresponding to various (excessive) power settings. The power settings cited are far above those appropriate for maximum endurance. For example, the Cessna 172 and 303 respectively require only 34 and 35% power at 2,000 feet and maximum gross weight. Since P_R decreases in proportion to $(\text{weight})^{3/2}$, required power settings drop below 30% when weights are less than 90% of maximum.

Maximum endurance simply means maximizing time aloft by reducing fuel flow to the minimum practical value. This is achieved by judicious choice of airspeed and power settings, as well as altitude, if practical. Maximum-endurance speed and power settings are always significantly below those for either maximum-range or long-range cruise.

It is well established (e.g., Hurt, 1965) that minimum thrust horsepower required, $(P_R)_{MIN}$, occurs at $.76V_{MR}$. It often is assumed that minimum brake horsepower and FF also coincides with $(P_R)_{MIN}$. Reduced powerplant efficiencies at such low power settings cause small differences in the speeds at which minimums in thrust horsepower and FF typically are reached. Equation 3b suggests maximum endurance/minimum fuel flow rate for a piston airplane occurs at $.78V_{LRC}$ or $.83V_{MR}$. This is the maximum endurance speed (V_{ME}) identified in Table 2. At this speed and power setting, FF is reduced to approximately 85% of that at V_{LRC}. Thus, fuel savings

Table 2
Speed Characteristics for Representative Airplanes

MODEL	NAME	V_S	V_{BGR}	V_{YSL}	V_{MR}	V_{LRC}	V_{ME}
C 172*	Skyhawk	52	66	75	82	88	68
PA 28	Cadet	56	71	81	82	88	68
TB 9	Tampico	58	71	80	85	91	71
AG 5B	Tiger	56	73	85	93	100	77
C172RG	Cutlass	57	74	84	96	103	80
M20J(ATS)	Mooney	59	89	86	106	113	88
A36	Bonanza	54	106	100	125	134	104
T 303	Crusader	68	105	104	114	122	95
B55	Baron	74	107	106	135	144	112
A-65-82	Queenair	83	103	109	119	127	99

* C 172 adjusted for speed fairings NOT installed.

can be substantial over a prolonged period of holding.

LEANING

It is appreciated that proper mixture control is critical to optimum cruise performance and engine longevity. Excellent coverage of the subject is readily available. Nevertheless, evidence suggests the subject could benefit from additional emphasis. When the airplane is paid for on a wet basis, incentives to lean properly are diminished. Handbook data show that failure to lean correctly can increase fuel consumption 20-25% above tabulated rates for a given power output. Obviously, waste of this scope negates gains achieved by skillful application of other cruise-control principles.

RPM VERSUS MAP

Piston-powered airplanes operate most efficiently at relatively low RPM and high MAP combinations for the given power requirement. Handbooks show specific range variations up to 7-8% between permissible RPM/MAP combinations. Therefore, once speed and power required are decided, that power should be produced at the most efficient permissible combination of low RPM and high MAP. This combination is not only the most fuel-efficient, but also the quietest and most conducive to engine longevity.

ILLUSTRATIVE EXAMPLES (C 172)

Examples of contrasts in cruise efficiency and the speed/fuel tradeoffs are shown in **Table 3**, using C 172 POH data (corrected for speed fairings NOT installed). The 300- and 500-mile legs are analyzed using the long-range cruise speed of 88 KCAS compared to a common combination of relatively high power/CAS and lower altitude. For simplicity, taxi fuel is not considered, and the combined climb and descent is assumed to produce an average specific range equivalent to the cruise components. Since all tabulated values are referenced to maximum gross weight, actual cruise fuel (for all legs) should be slightly less than tabulated values. However, the comparative relationships between efficient and inefficient profiles would be preserved.

For leg A (300 NM), use of 75% power at 4,000 feet required 16% (3.1 gallons) greater cruise fuel, while saving only 18 minutes (10%) of time, compared to efficient cruise at V_{LRC} and 10,000 feet. For leg B (500 NM), the contrast is more dramatic. The low

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Table 3
Cross-Country Illustration (C 172)

LEG	DIST	ALT	TAS	CAS	% POWER	TIME	CR FUEL
A1	300	4,000	114	107	75	2h 38m	22.0
A2	300	10,000	102	88	57	2h 56m	18.9
B1	50	4,000	114	107	75	4h 23m++	36.7++
B2	500	10,000	102	88	57	4h 54m	31.5

altitude/high power combination (B1) would provide insufficient reserve without a fuel stop, whereas the long-range cruise combination (B2) could safely avoid a stop. The latter would save 5.2 gallons of fuel, plus that required for maneuver and taxi incident to the refueling stop. Furthermore, it is unlikely that the total refueling delay would not exceed the nominal 31-minute difference between legs B1 and B2 and would save about 8 gallons (21%) of fuel in the process.

HOW TO FIND AN AIRPLANE'S MAXIMUM RANGE AND ENDURANCE AIRSPEEDS

The importance of proper speed (CAS) selection at any altitude is inarguable if high cruise efficiency is to be achieved. Perhaps the pilot will find that the data in Table 2 provide an adequate estimate for the airplane in question.

Airspeeds for maximum range and best power off glide often are assumed to be identical, since theoretically both occur at $(L/D)_{MAX}$. However, actual ratios of V_{MR}/V_{BGR} in Table 2 range between 1.1 and 1.3, and average 1.22. In addition to powerplant efficiency variations discussed previously, this disparity is related to the increased drag and reduced airspeed for $(L/D)_{MAX}$ with propellers either windmilling or feathered.

Careful analysis of Table 2 data provided no satisfactory correlation between V_{MR} and POH values for stall, best glide, or best rate-of-climb speeds. For the samples in Table 2, Kershner's rules of thumb (1985) worked reasonably well for all except the Bonanza and Queenair, yielding an average difference in V_{MR} of 4 knots for the other eight airplanes. Kershner's rules of thumb "are not intended in any way to replace the figures as given by the POH or other comparable information sources, if available." (p. 4) The following methods are model-specific, and should be tested in as many ways as possible, including use of Kershner's rules.

Despite the frustrations described above, there are two other independent methods for estimating V_{MR} and V_{LRC}. One is based on POH data and the other on simple in-flight measurements.

Method 1: Using POH Data to Find V_{MR}

As previously noted, the POH will not supply V_{MR} or V_{ME} explicitly. However, for most handbooks it is possible to use the cruise range data to estimate these critical speeds with reasonable accuracy. The following steps have been used successfully:

1. Examine POH cruise data, beginning at the higher tabulated altitudes, since low altitudes will not bracket V_{MR}(CAS). The 10,000- or 12,000-foot tabulations are a good starting point for unpressurized airplanes, with 20,000 or above for pressurized airplanes. For simplicity, use the standard (ISA) temperature values so that pressure and density altitudes coincide. Note the reference weight used.

2. Use the POH cruise data to convert tabulated TAS and fuel flow (either GPH or PPH) to specific range (SR). Divide TAS by its corresponding FF to obtain SR, either in NM per gallon or NM per pound. SR units are optional, provided consistency is maintained.

3. Locate the peak SR value, which defines V_{MR}. A simple plot of SR versus TAS is very helpful. If possible, repeat this procedure to find V_{MR} at other altitudes.

4. The TAS value found depends upon density altitude and SMOE, the ratio between TAS and CAS. Conversely, the desired V_{MR} (CAS) result is independent of altitude. Therefore, the TAS results from step 3 must

be converted to CAS by dividing by SMOE. Table 4 provides SMOE versus density altitude.

5. Use the average of the CAS values from step 4 as the best estimate of V_{MR}. Small variations between altitudes should not cause concern, due to the shape of the curve near its peak. Review Figures 3, 4, and 5.

6. Use the V_{MR}(CAS) so determined to establish V_{LRC} and V_{ME}. $V_{LRC}=1.07V_{MR}$ and $V_{ME}=0.83V_{MR}$.

7. The speeds in step 6 correspond to the POH reference weight, normally gross weight. They must be adjusted for lower weights, in proportion to the square root of weight, as illustrated in Figure 1. This process maintains constant AOA flight conditions, e.g., V_{LRC}, as weight decreases. Tabulate or plot CAS versus weight across the normal range of cruise weights. As a final step, use the airspeed calibration data to convert CAS to IAS for ready reference in flight. A C 172 example is illustrated in **Table 5**, where an IAS reduction of 2 knots per 100 pounds provides the desired constant AOA, long-range cruise profile.

Method 2: In-Flight Measurement of V_{ME}

Direct in-flight measurement of V_{MR} is complex, while measurement of V_{ME} simply requires measurement of the airspeed corresponding to minimum engine power (eg., RPM) required for stabilized, constant altitude flight. Assuming the same speed for minimum BHP and THP, the ratio of $V_{ME}/V_{MR}=0.76$ allows indirect determination of V_{MR} and V_{LRC}, by measuring V_{ME}.

Under the author's supervision, this procedure was used at Embry-Riddle Aeronautical University for 5 of the 10 airplane models listed in Table 2. Three individual tail numbers were used for each model, and average test values established. V_{ME}, the IAS at minimum power, was measured in flight, then converted to CAS. The result was divided by .76 to yield V_{MR} for test weight. Finally, V_{MR} was corrected to the standard gross weight by multiplying by (gross weight/test weight)$^{0.5}$. Results from this method bracketed the values found using the POH method. Two were higher, two lower, and one agreed perfectly. The differences averaged only 3 knots and are considered practically negligible.

GENERAL FLIGHT PLANNING METHODOLOGY

The foregoing principles can be applied directly to establish the best balance among the five cruise variables of power, altitude, speed, weight, and wind. Baseline cases of maximum speed/minimum time and long-range cruise will be examined.

Table 4
SMOE versus Altitude

DENSITY ALTITUDE	SMOE= TAS/CAS
SL	1.000
1,000	1.015
2,000	1.030
3,000	1.045
4,000	1.061
5,000	1.077
6,000	1.094
7,000	1.111
8,000	1.128
9,000	1.146
10,000	1.164
11,000	1.182
12,000	1.201
15,000	1.261
20,000	1.370

Case 1: Maximum Speed/Minimum Time

This case emphasizes minimum time enroute over other considerations, and the leg is assumed sufficiently short that high fuel consumption will not occasion an extra stop. Since the airplane's maximum sustainable ground speed is sought, the steps are:

1. Select the maximum continuous cruise power setting.

2. Determine the available altitude providing greatest TAS for the selected power setting. (Ordinarily the optimum no-wind altitude corresponds to the maximum altitude at which the desired power setting can be maintained, around 8,000 feet in the case of 75% power for naturally aspirated engines.)

3. Determine if wind shear is significant; if it is, trial and error may improve on the no-wind (POH) altitude choice from step 2, since the altitude yielding greatest ground speed is sought.

4. Use the V/V_{MR} ratio and Figure 5 to estimate the resulting penalty from maximum range.

Carson (1982) asserted that the least wasteful way of wasting fuel considers the airplane's inherent advantage

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as a time-saver and occurs where the product of V(L/D) is a maximum. This speed produces the optimum ratio of (TAS/FF), and corresponds to $1.32V_{MR}$ when powerplant efficiency is independent of speed. Modification of this ratio to incorporate the characteristics of Figure 5 and Equation 3a suggests the "least wasteful" speed actually is nearer $1.21V_{MR}$. This condition is approximately 12% quicker and 8% less range efficient than Case 2 below.

Table 5
C 172 Cruise Profile

C 172 CAS/IAS						
WT	2,400	2,300	2,200	2,100	2,000	1,900
V_{LRC}	88/89	86/87	84/85	82/83	80/81	78/79

Case 2: Long Range Cruise (99% Maximum Range)

This case emphasizes range efficiency over time enroute. It can be modified to V_{MR} and R_{MAX} in situations where available fuel is marginal and the additional 1% saving is significant. The steps are:

1. Since aerodynamic and powerplant efficiency is practically independent of altitude, select the available cruise altitude which provides most favorable winds (least headwind/greatest tailwind). Significant enroute wind shifts require separation of cruise parameters into multiple segments to optimize total trip efficiency.

2. Select the CAS and IAS for V_{LRC} or V_{MR}, as appropriate, corresponding to initial cruise weight.

3. In case of a headwind exceeding 25% of the resulting TAS, or a tailwind of any amount, adjust TAS and CAS using the rules provided.

4. Select the most economical power setting which provides the desired CAS, by optimizing MAP/RPM and mixture settings.

5. Periodically reduce power to maintain CAS proportional to $(\text{weight})^{0.5}$ and constant optimum cruise angle of attack. The required speed reduction is about .7% per hour for single-engine/naturally aspirated airplanes and 1.3% per hour for turbocharged airplanes. The rate of power and fuel flow reduction is triple the slight speed-reduction rate, hence the longer the flight, the more significant the economies from properly adjusting power and speed.

6. Should wind shear be negligible and TAS slower than desired, elect a higher altitude to increase SMOE and TAS approximately 1.5% per 1,000 feet. Note that the CAS rules in steps 2-5 are independent of altitude.

ANSWERS TO VARIABLE ALTITUDE EXERCISE: 1, 5, 6, and 11 are "S."□

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